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Deflection of Mesons in Magnetized Iron

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(Received May 8, 1944)

The deflection of mesons in a magnetized ferromagnetic medium was investigated. A beam of mesons was made to pass through 9 cm of iron, and the resulting distribution of the beam was observed. Two arrangements were employed. In the first arrangement, the deflection due to the field caused a fraction of the mesons to hit a counter placed out of line with the others. An increase of sixty percent in the number of coincidences was recorded when the iron was magnetized. In the second arrangement, all the counters were arranged in line, and the deflection due to the field caused an eight percent decrease in the number of coincidences. These results are compared with theoretical predictions deduced from the known momentum spectrum of the mesons and from the geometry of the arrangement. The observed effects agree as well as can be expected with those calculated under the assumptions that the effective vector inside the ferromagnetic medium is the induction B, and that the number of low energy mesons is correctly given by the range-momentum relation.

INTRODUCTION

THE idea of observing the deflection of a cosmic-ray particle passing through a magnetized ferromagnetic medium was independently suggested and carried out experimentally by Rossi[1] and Mott-Smith.[2] Both these experimenters obtained negative results, which were interpreted at the time as meaning that the energy of the cosmic-ray particles was higher than expected.

These experiments provoked a discussion among theoretical physicists about the effective vector acting on a charged particle moving through a ferromagnetic medium. The classical theory gives a result that depends on the particular model chosen for the elementary magnets.[3] Moreover, one can hardly rely on the results of the classical theory since ferromagnetism is due to the electron spin and hence is a typically nonclassical effect. Fortunately, a calculation based on Dirac's theory of the electron has been performed by Weizsaecker,[4] the unambiguous result being that the effective vector acting on the particle is the magnetic induction B.

Further experiments were performed by Alvarez,[5] who investigated the deflections of β-particles passing through a thin magnetized iron sheet. The observed effect was much smaller than expected, and he concluded that the effective vector is at most of the order of one-tenth of B.

Danforth and Swann[6] performed another experiment on cosmic rays. Their counters were

[1] B. Rossi, Nature **128**, 300 (1931); Lincei Rend. **2**, 478 (1930).
[2] L. M. Mott-Smith, Phys. Rev. **37**, 1001 (1931); **39**, 403 (1932).
[3] W. F. G. Swann, Phys. Rev. **49**, 574 (1936).
[4] C. F. v. Weizsaecker, Ann. d. Physik **17**, 869 (1933).
[5] L. Alvarez, Phys. Rev. **45**, 225 (1934).
[6] W. E. Danforth and W. F. G. Swann, Phys. Rev. **49**, 582 (1936).

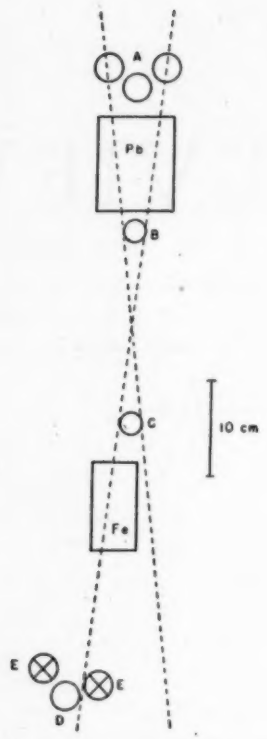

FIG. 1. Counter arrangement. Counter D is represented in the position outside of the beam defined by counters B and C. Cross-marked counters are used as anticoincidence counters.

arranged in a straight line, an iron block being placed between the two last counters. A decrease of the order of ten percent in the number of coincidences was observed when the iron was magnetized. This result was interpreted, on the basis of the then current ideas about the nature and energy distribution of the cosmic-ray particles, as meaning that the effective field was of the order of B.

The preceding summary of the results of earlier experimenters shows that it was worth while to undertake a new investigation of the problem in view of the improved technique now available. Moreover, the present knowledge of the nature of cosmic-ray particles at sea level affords an unambiguous interpretation of the results.

EXPERIMENTAL ARRANGEMENT AND RESULTS

In order to obtain results which admit of a clear interpretation, only one kind of particle must be selected. Moreover, radiation processes and the consequent cascade multiplication make the soft component wholly unfit for experiments of this sort. Therefore, one must employ a beam of mesons selected by preliminary absorption of the soft component.

The choice of the optimum thickness of magnetized iron to be employed requires some consideration. For a given thickness l, there is a minimum value of the momentum $p_{\min}$ of the mesons that are able to traverse the iron. For a given momentum p, and hence for a given radius of curvature R of the path, the deflection will be approximately proportional to l, and inversely proportional to p. Were the momentum proportional to the range, the maximum angular deflection obtainable would be independent of the thickness. However, in the low energy region the range increases rapidly with momentum, and it is only in the region of highly relativistic energies that the range-momentum relation becomes linear. Consequently, the thickness of iron must correspond to particles whose kinetic energy is at least as large as the self-energy. On the other hand, the use of an excessive thickness of iron would excessively reduce the intensity of the beam, owing to geometrical factors. When all these factors are considered, it is seen that the most favorable thickness of iron is of the order of ten cm. In the actual experiments, a thickness of 9 cm was employed, corresponding[7] to a minimum momentum of 2.1×10^8 ev/c (throughout this paper, the unit of momentum will be 10^8 ev/c).

Due allowance must be made for the scattering of mesons by the iron. Since the average angle for multiple Coulomb scattering and the magnetic deflection are, for relativistic energies, both inversely proportional to the energy, scattering will equally affect the results at all energies, provided these are sufficiently high. Hence the effect of scattering, although important, does not appreciably affect the choice of the optimum thickness.

The arrangement of the counters is shown in Fig. 1. Counters A, B, and C, with the interposed ten cm of lead, select a beam of mesons. A fourth counter D can be placed either in line with the other three, or wholly outside of the beam defined by the coincidences (BC). The iron block can be placed between the counters C and D. Two

[7] B. Rossi and K. Greisen, Rev. Mod. Phys. 13, 240 (1941).

anticoincidence counters E are placed one on each side of the counter D, the purpose being to discriminate against showers that might give fourfold coincidences ($ABCD$) and against particles that, traversing the iron, produce secondaries. By displacing the counter D, the definition of the meson beam was tested. It was found that when the counter D was just outside of the beam, the number of fourfold coincidences dropped to about one percent of the number recorded with the four counters in line. This very low "background" is mainly responsible for the success of that part of the experiment in which the counters were placed out of line and was made possible by the use of anticoincidence counters.

With counter D out of line, the number of fourfold coincidences was almost doubled when the iron block was interposed, owing to scattering by the iron. This effect, however, did not prevent the measurement of the deflection due to the magnetic field, but made it necessary to extend the measurements over a longer period. The readings were alternately taken with and without field, and the difference was assumed to be due to the effect of the field.

The iron block, 9×4.5 cm^2 in cross section and 25 cm in length, constituted part of a closed magnetic circuit and was kept magnetized at 15,000 gauss by an electromagnet dissipating 40 watts.

Counters A, D, and E had an effective diameter of 2.9 cm and an effective length of 37 cm. Counters B and C had an effective diameter of 2.3 cm and an effective length of 22 cm. The distances between the counters are shown to scale in Fig. 1.

TABLE I. Fourfold coincidences with magnetized and unmagnetized iron. Counters out of line.

Magnetic induction B	Number of hours	Number of coincidences	Coincidences per hour
0	1200	1186	0.99±0.03
+15,000	650	1073	1.65±0.05
−15,000	650	1028	1.58±0.05

The results obtained with the counters out of line are summarized in Table I. All the counts were taken with counter D placed 0.4 cm outside of the beam defined by the threefold coincidences (ABC). It is seen that the magnetic field increases the number of counts by about sixty percent, a far larger effect than any previous experimenter has been able to obtain. The sign of the field is indicated as positive when the positively charged particles are deflected into counter D. The difference between the numbers of positive and negative particles is within the statistical error. There is, however, a slight indication of an excess of positive mesons, as reported in the literature.[8]

TABLE II. Fourfold coincidences with magnetized and unmagnetized iron. Counters in line.

Magnetic induction B	Number of hours	Number of coincidences	Coincidences per hour	Difference per hour	Difference percent
0	450	13321	29.6±0.26		
15,000	450	12368	27.5±0.25	2.1±0.36	7.1±1.2

The experiment with the counters in line simply consisted in counting the fourfold coincidences with and without magnetic field. Obviously, in this case there is no point in reversing the field, as the arrangement is symmetrical with respect to positive and negative mesons. The decrease in the number of fourfold coincidences due to the field was of the same order of magnitude as that observed by Danforth and Swann. The results are summarized in Table II.

INTERPRETATION OF RESULTS

We shall first discuss the results obtained with the counters out of line.

In order to compare the experimental results with the theory, one must know the differential momentum spectrum of the mesons $f(p)$ and the probability $w(p)$ that a meson recorded by counters B and C will be deflected into counter D by the action of the field, assuming that the induction B is effective. Let the function $f(p)$ be normalized so that

$$\int_0^\infty f(p)dp=1,$$

then the ratio of the number of fourfold coincidences due to the field to the number of incident mesons [coincidences (BC)] will obviously be

[8] H. Jones, Rev. Mod. Phys. 11, 235 (1939).

given by:

$$n=\int_{p\min}^{\infty} w(p)f(p)dp.$$

The spectral distribution $f(p)$ can be taken from data available in the literature, and the probability $w(p)$ can be calculated from the geometry of the experiment. To perform this calculation, we shall make the following simplifications: (a) We shall assume that the meson spectrum is not appreciably affected by filtering through the roof and two floors of the building, as well as through the lead block. The distortion in the energy distribution thus introduced is certainly small, and probably within the limits of accuracy of the present knowledge of the spectrum. (b) The deflections due to scattering and to the magnetic field in the iron block are assumed to be independent of each other; that is, the effect of the field is obtained by subtracting the background observed with the unmagnetized iron. Actually, scattering will somewhat alter the intensity distribution in the undeflected beam, especially for low momentum mesons that are mainly responsible for the magnetic effect. This is believed to represent the main source of error in comparing the experimental results with the theory. (c) The radius of curvature of the meson path varies as the meson loses energy in the iron. To simplify the calculations, an average radius will be substituted.

The momentum spectrum of the mesons is wholly determined, for low energies, by the range-momentum relation. This relation, if we assume that the whole energy loss is due to collisions, is well known theoretically. The resulting momentum spectrum has been calculated and represented graphically, with an approximation sufficient for our purposes, by Euler and Heisenberg.[9] The spectrum at high energies has been deduced by several experimenters from the measurement of the curvature of tracks in the cloud chamber. A composite spectrum was obtained by joining, at $p=14$, the theoretical curve of Euler and Heisenberg with the experimental data of Jones.[8] This procedure was necessary in order to normalize the function $f(p)$ although mesons with momentum larger than 14 are ineffective in the present experiment. The differential spectrum used is shown graphically in Fig. 2.

The probability $w(p)$ for a meson recorded by counters B and C to be deflected into counter D was calculated as follows. The intensity distribution of the undeflected beam in the plane of the counter D is readily evaluated geometrically. To a first approximation, the effect of the field on the particles of one sign can be taken as a shift of the whole beam toward one side, by an amount inversely proportional to the radius of curvature and hence to the momentum. The fraction of the shifted beam that passes through counter D is calculated.

The function $w(p)$ is evaluated for p between 0 and 14 (for higher values of p the probability

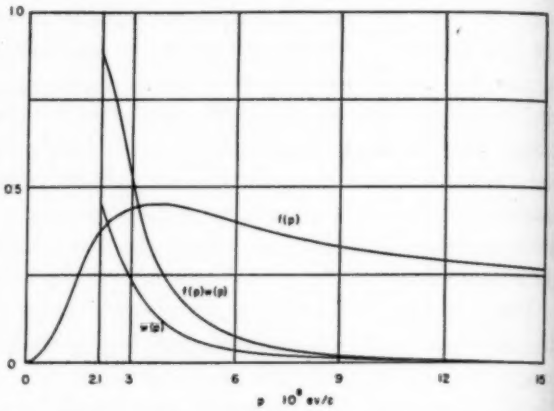

FIG. 2. $f(p)$, differential momentum spectrum; multiply ordinates by 0.1. $w(p)$, probability for a meson to be deflected into counter D. $f(p)w(p)$, product of the two functions; multiply ordinates by 0.02.

is practically zero) and is represented in Fig. 2. The figure also represents the product $f(p)w(p)$; from this latter curve it is seen that most of the contribution to the magnetic effect comes from mesons that have little more than the energy necessary to traverse the iron block.

A numerical integration gives:

$$n=\int_{2.1}^{14} w(p)f(p)dp=0.028.$$

In order to compare this figure with the experimental data, it is also necessary to know what fraction N of the beam defined by the coincidences (BC) passes through counter D when the latter is in line with the other two. This

[9] H. Euler and W. Heisenberg, Ergeb. d. exakt. Naturwiss. 17, 1 (1938).

fraction can immediately be evaluated from the geometry and is 0.49. The theoretical value of the ratio n/N is therefore:

$$(n/N)_{\text{theor}}=0.057.$$

Since only mesons of one sign can be deflected into counter D, we shall take the sum of the effects due to the positive and negative fields after subtracting the effect measured without field. This sum is 1.25 ± 0.08 coincidences per hour. The number of fourfold coincidences with the counters in line is 29.6 per hour. Hence

$$(n/N)_{\text{exper}}=1.25/29.6=0.042\pm0.003.$$

Considering the uncertain factors involved in the calculations, we may state that there is no significant discrepancy between the observed and the calculated effect.

We shall now consider the experiment with the counters in line. The calculations are similar to those discussed in the preceding case; hence the procedure followed will be briefly outlined.

In this case, the displacement of the beam due to the field gives rise to a decrease in the number of particles recorded by counter D. This decrease was calculated, from the geometry of the beam, as a function of momentum, and a numerical integral was again evaluated on the basis of the same spectral distribution as before. The theoretical result is that the field should produce a decrease of 8 percent in the number of fourfold coincidences. From Table II it is seen that the observed effect is 7.1 ± 1.2 percent. Hence, the experimental and theoretical values agree to within the statistical error.

We now wish to discuss the significance of the present experiments. The agreement between the calculated and the observed effects was obtained under two assumptions: (a) the effective vector acting on the particles inside the magnetized medium is the induction B; (b) the number of low energy mesons is correctly given by the range-momentum relation. Were the effective field considerably smaller than B, or were the number of low energy mesons smaller than assumed, then the observed effect would not have agreed with the theoretical predictions.

It would at first appear that the present experiment could not be considered as a test of both of the above-mentioned assumptions. We must consider, however, that if either of the two determining factors disagreed with the theoretical predictions, this disagreement would be only in one direction. The effective vector acting on a particle could conceivably be smaller than B (under certain classical assumptions, for example, one obtains values that are intermediate between H and B); but one could hardly imagine a reasonable hypothesis under which the effective vector could be larger than B. Regarding the number of low energy mesons, here again one may readily imagine mechanisms[10] capable of removing mesons from the beam, and hence of rendering the number of slow mesons smaller than assumed; but it would be difficult to conceive of an effect that could make the number of slow mesons higher than calculated from the range-momentum relation. We conclude that the agreement of the experimental results with the theory is a good proof, at least, of the approximate validity of both hypotheses, namely, that the effective vector is B and that the number of slow mesons is correctly given by the range-momentum relation.

[10] An interaction that might produce this effect has been discussed by J. Hamilton, W. Heitler, and H. W. Peng, Phys. Rev. 64, 78 (1943). Their theoretical conclusions do not seem to be supported by the results of the present experiments.

PHYSICAL REVIEW VOLUME 66, NUMBERS 1 AND 2 JULY 1 AND 15, 1944

The Effects of Various Ions on the Infra-Red Absorption of Water

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AND

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(Received May 1, 1944)

The effects of various non-hydrolyzing salts on the infra-red spectrum of water have been studied in the region between 1μ and 7μ. Variations in the positions of the absorption maxima at 1.46μ, 1.79μ, 1.96μ, 3.0μ, 4.7μ, and 6.1μ have been observed. The changes produced by the solutes have been compared with existing data on the effects of temperature variation on the absorption of water. The results indicate that, in general, large ions produce changes similar to the changes produced by increasing the temperature of the pure solvent, whereas small ions produce the reverse effect. These results are in agreement with data on the other physical properties of water and aqueous solutions.

ON the basis of x-ray diffraction data, Bernal and Fowler[1] have developed a theory of the intermolecular structure of water and aqueous solutions. According to this theory, the molecules of water in the liquid state are arranged in local crystalline lattices. The extent of these lattices and the regularity of the arrangement of the molecules within the lattices are greatest at low temperatures. The effects of the presence of ions in the water depend upon the polarizing powers of the ions, polarizing power being defined as the ratio of ionic charge to ionic radius. In a sequence of ions of equal charge, the larger ions decrease the regularity of molecular arrangement and produce changes similar to those produced by elevation in the temperature of pure water; smaller ions tend to enter into the crystalline lattices and preserve regularity.

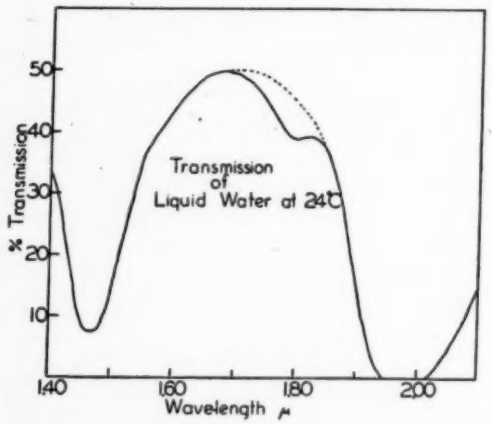

FIG. 1. The transmission spectrum of water in the vicinity of the 1.79-μ band.

As the infra-red absorption bands of water have been found to change in position as the temperature is varied,[2,3] it has been of interest to study the influence of various ions on these bands in order to compare the observed effects with the effects predicted on the basis of the Bernal-Fowler theory. Such studies have already been made for the 4.7μ band.[4,5] The purpose of the present investigation has been to extend these studies to include all the major infra-red bands between 1μ and 7μ.

The spectrometer used was an instrument of the Wadsworth-Littrow type equipped with a large glass prism for use between 1μ and 2μ and a rocksalt prism for use at longer wave-lengths. Absorption cells with glass windows were used at wave-lengths shorter than 2μ and fluorite cells were used between 2μ and 7μ. Cell thicknesses were adjusted to give optimum measurable absorption for each band. The bands studied were located at 1.46μ, 1.79μ, 1.96μ, 3.0μ, 4.7μ, and 6.1μ. Assignment of these bands to various types of molecular vibration have been made by earlier investigators.[6]

The compounds chosen for study were the alkali and alkaline-earth halides.[7] The ions pro-

[1] J. D. Bernal and R. H. Fowler, J. Chem. Phys. 1, 515 (1933).

[2] J. R. Collins, Phys. Rev. 26, 771 (1925).

[3] E. Ganz, Ann. d. Physik 28, 445 (1937).

[4] E. K. Plyler and E. S. Barr, J. Chem. Phys. 6, 316 (1938).

[5] D. Williams and T. N. Gautier, Phys. Rev. 56, 616 (1932).

[6] J. W. Ellis, Phys. Rev. 38, 693 (1931).

[7] All compounds were obtained from Eimer and Amend;

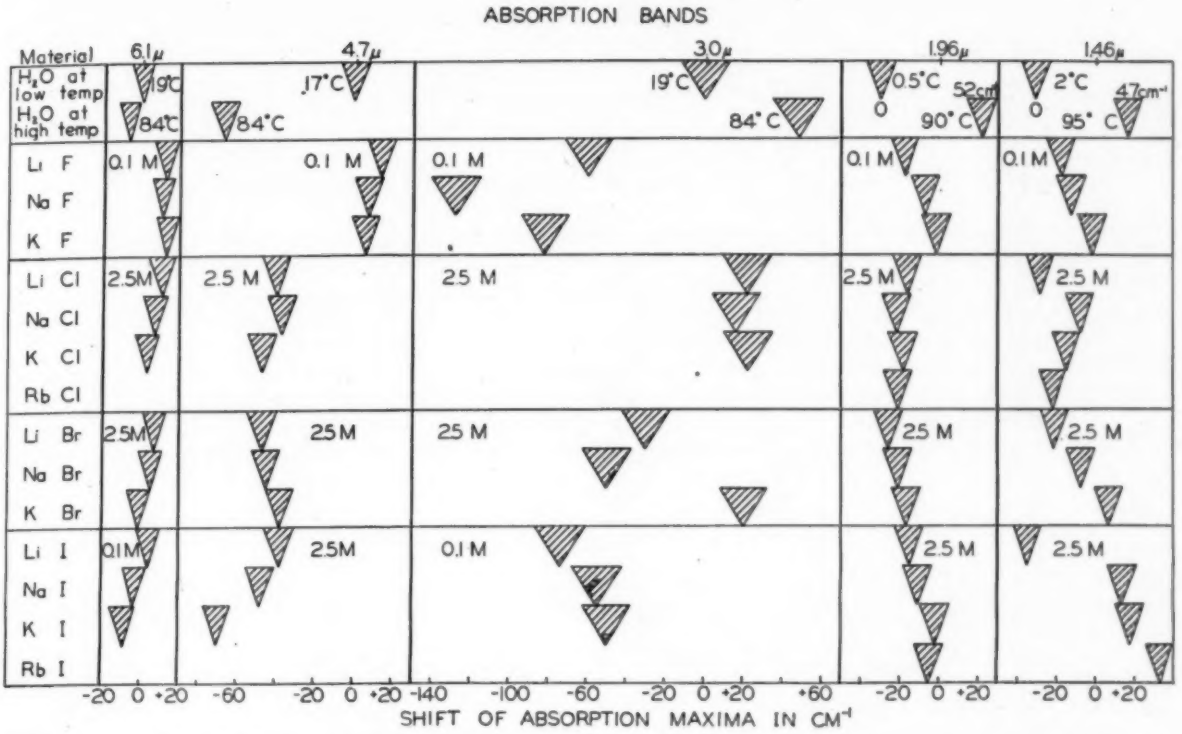

FIG. 2. Changes in the characteristic vibrational frequencies of water: The plots show frequency changes produced by various positive ions in the presence of common negative ions.

duced are monatomic and hence have no absorption in the region being investigated. In order to determine the effects of various cations on a given band, a sequence of solutions of compounds containing these cations in combination with a common anion was studied. The reverse procedure was used to ascertain the effects of various anions. The same molar concentration was used for each solution in a given sequence. In general, the concentration was made as large as possible, but it was found desirable to use less concentrated solutions in the thin cells employed for the 3.0-μ and 6.1-μ bands.

Definitely measurable shifts produced by solutes were observed for all bands except the weak band at 1.79μ. The absorption of water in this region is shown in Fig. 1. It will be observed from this figure that the 1.79-μ band appears as a small depression, on the side of the intense band with maximum at 1.96μ. As indicated by Collins,[8] the location of the 1.79-μ maximum can be determined satisfactorily by taking the ratio of the observed transmission to an assumed background such as that given by the dotted curve in Fig. 1. However, the background absorption changes when solutes are introduced, since the solutes produce changes in the intense 1.96-μ band. Hence, it is necessary to assume a different background for every solution studied and it is felt that the shifts detected by such methods are not sufficiently quantitative to warrant publication. However, it can be asserted that the changes produced by the fluoride and iodide ions are of the type to be expected from the Bernal-Fowler theory. No definite statements can be made about the effects of ions of intermediate size.

The results obtained for the other bands are summarized in Figs. 2 and 3. In these figures the shifts of absorption maxima are plotted. The observed shifts are indicated by the vertices of the shaded triangles and the uncertainties in the location of the maxima are indicated by the length of the bases of the triangles. Concentrations are given for each sequence of solutions.

only small quantities of the cesium and rubidium compounds were available and these were used for work in the short wave-length region.

[8] J. R Collins, Phys. Rev. 52, 88 (1937).

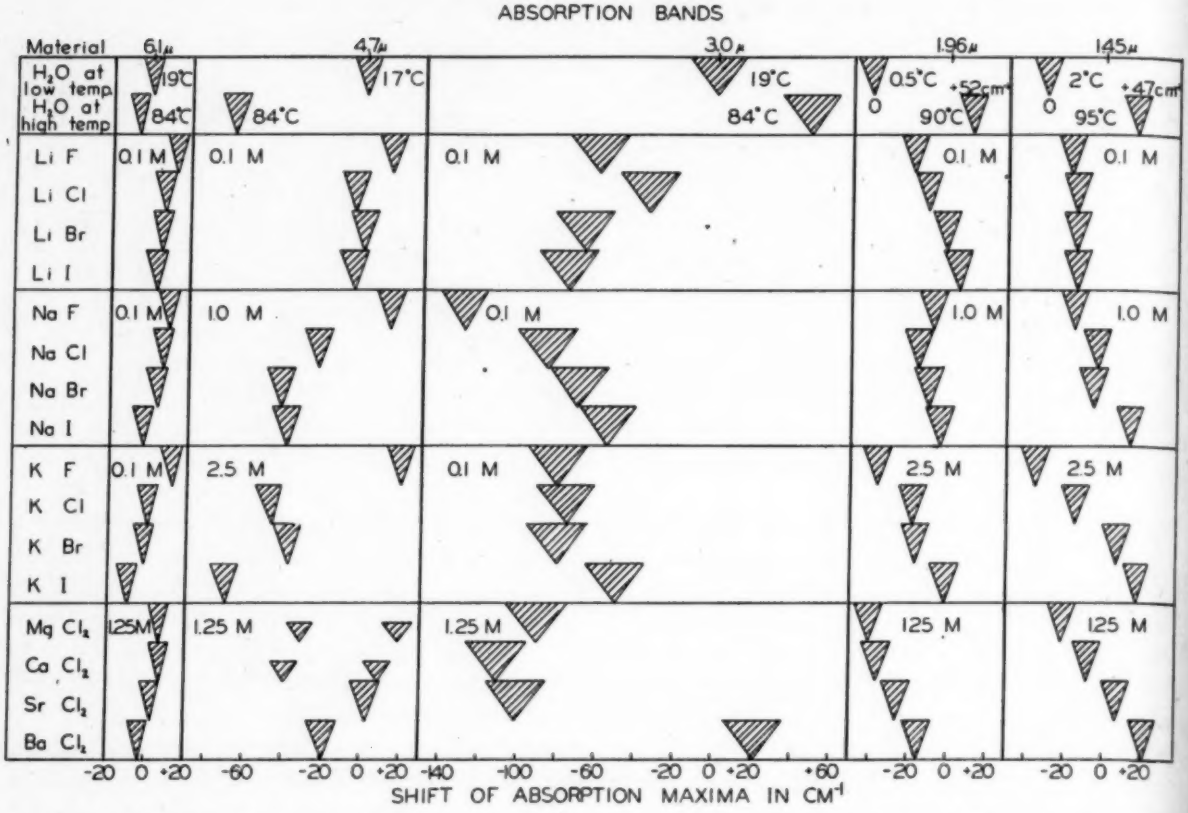

FIG. 3. Changes in the characteristic vibration frequencies of water: The plots show frequency changes produced by various negative ions in the presence of common positive ions.

The data for water at various temperatures are taken from Ganz' work on the 6.1-μ, 4.7-μ, and 3.0-μ bands and Collins' work on the 1.96-μ and 1.46-μ bands.

Figure 2 gives the shifts produced by various positive ions in the presence of common negative ions. According to Bernal and Fowler, increasing ion size should produce effects similar to those produced by increasing the temperature. Of the 20 sequences summarized in Fig. 2, 14 sequences exhibit shifts of the type predicted from the theory, 3 sequences show shifts in the opposite direction, and 3 sequences show no definite trend.

In Fig. 3 are shown the shifts produced by various negative ions in the presence of common positive ions. Of the sequences shown, 16 show shifts of the predicted type, one has a shift to the opposite direction, and three show no definite shift. It should be noted that $MgCl_2$ and $CaCl_2$ produce a definite splitting of the 4.7-μ band.

Six of the sequences for which the shifts were not in agreement with those predicted from the theory were for data obtained in the 3-μ region. The broad band in this region is produced by three overlapping bands. Hence, it is probably desirable to repeat the measurements in this region with an instrument of higher resolving power and with thinner absorbing layers than could be obtained with the type of cells used in this study. It would also be desirable to make a thorough study of the effects of concentration on the changes produced by the solutes.

The results obtained are in general agreement with the predictions made on the basis of the Bernal-Fowler theory. However, the fact that there are some exceptions probably indicates that there are factors other than the polarizing power of the ions which should be considered in developing a more complete theory.

The writers wish to express their appreciation to Miss Annette Herald for preparation of the figures.

The Effect of Adsorbed Gases on Contact Electrification*

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(Received May 25, 1944)

The magnitude of the electrification produced on contact of quartz and of sodium chloride on nickel in atmospheres of air, oxygen, and nitrogen has been measured. Reproducible pressure-dependent results were obtained that indicate the contact electrification is a surface phenomenon and depends greatly on the nature of the surfaces involved. At least two layers of adsorbed gas play an important part in the phenomenon of contact electrification. In the pressure range from 760 to 30 mm at least one layer of adsorbed gas is removed. In this pressure range the surface covered with adsorbed gas makes the major contribution to the charge separation. At pressures below 0.1 mm at least a second layer of adsorbed gas is removed. In this pressure range the major contribution to the charge separation is made by the surface from which the second layer of gas has been removed. A fit of the data to Langmuir adsorption isotherms was obtained. Water vapor and perhaps hydrogen act as surface poisons in the phenomenon of contact electrification.

CONTACT or frictional electrification and contact potential difference are two "solid to solid" contact phenomena that lead to the electrical charging of the solids involved. Contact potentials are observed only when both of the solids are metals. This effect may be readily explained in terms of the electron theory of metals. The magnitude of the contact potential may either be measured directly or computed from the work functions of the metals. The results thus obtained are consistent and reproducible.

When one or both of the solids is a non-conductor, the measurement of contact potential is not possible or rather meaningless. Non-conductors do not accept or give up electrons readily. Such bodies do not have the free electrons which are necessary for the phenomenon of contact potential difference. Therefore the theory which explains contact potentials cannot explain contact electrification. Nor has there yet been published a theory which satisfactorily explains this phenomenon.

The charging of two non-conductors by friction was the first form of electrical charging discovered. The phenomenon of charging by impact or contact is a very widespread one encountered in dust storms and in many industrial processes where there is friction between non-conductors. The control and applications of contact electrification in industry would be of greater importance if the phenomenon were better understood. While a great deal of work has been done, very little satisfactory information has been obtained. Nearly all the published data are qualitative and often are contradictory and not reproducible.[1–5] In fact the very nature of the phenomenon is obscure. It was suggested by Professor Loeb that this phenomenon be further investigated and that great care be taken to work with reproducible surfaces in a reproducible environment with particular reference to the nature and pressure of the gases in contact with the surfaces.

Nearly every man who has worked on the contact potentials of metals has noticed and remarked on the effect of adsorbed gases on the potentials measured. It is now customary to take great pains to remove all traces of gas, to prepare clean surfaces, before making a measurement of contact potential.

Recently the importance of adsorbed gas in another contact phenomena, friction, has been shown.[6,7] It therefore seems strange that the few men who have studied contact electrification

* Submitted to the faculty of the University of California in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[1] P. E. Shew and C. S. Jex, Proc. Roy. Soc. London **A118**, 97–113 (1928).
[2] A. Fleming, Proc. Roy. Soc. London **53**, 51–3 (1941).
[3] R. Schnurman, Proc. Phys. Soc. London **53**, 547–53 (1941).
[4] P. Boning, Zeits. f. tech. Physik **8**, 385–98 (1927).
[5] H. Israel, Zeits. f. tech. Physik **9**, 289–93 (1928).
[6] R. Schnurman, Proc. Phys. Soc. **53**, 538–46 (1941).
[7] H. Donandt, Chem. Abs. **36**, 3466 (1942).

recently have paid little or no attention to the gases adsorbed by the solids used.

I have attempted to study quantitatively the charge separation produced on the contact of two dissimilar solids, one of them a non-metallic, and the effect of gas pressure on this charge separation.

APPARATUS

The apparatus consists of two nickel flasks mounted coaxially and separated by a cylindrical insulator made of quartz. One of the flasks is filled with small particles of the non-metallic solid. The end plates of the flasks are perforated so that equilibrium gas pressure may be quickly established. The whole is enclosed in a glass envelope connected to a vacuum system through a ground glass joint. (See Fig. 1 for a cross section of the apparatus.)

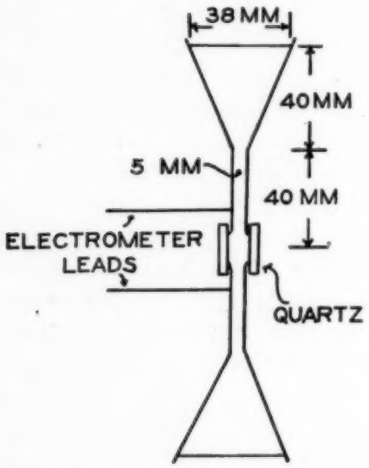

FIG. 1. Cross section of the apparatus.

When the envelope is inverted, the non-metallic particles fall from one flask to the other, carrying with them a charge. The potential built up between the flasks is measured with a quadrant electrometer. From this potential and from the known capacity of the apparatus the charge separation is computed.

The total capacity of the system, flasks, electrometer, and parallel capacitor is 750$\mu\mu$f.

MATERIALS

The quartz particles used were prepared by crushing amorphous quartz in an agate mortar. The NaCl was prepared from commercial salt. A hot saturated solution of commercial salt was filtered and then heated until about one-fourth of the water had evaporated. The crystals formed were recovered, dried, and crushed in the agate mortar.

TABLE I. Physical constants of the particles used.

	Quartz	Quartz	Quartz	NaCl
Weight	7.02 g	4.74 g	7.81 g	8.10 g
Density	1.55	1.55	1.55	2.17
Total vol. (cc) of non-metallic used	4.52	3.05	5.03	3.73
Av. vol. (cc) of individual particle	3.2×10^{-6}	3.2×10^{-6}	3.2×10^{-6}	3.2×10^{-6}
Number of particles	1.41×10^{6}	0.95×10^{6}	1.57×10^{6}	1.16×10^{6}

TABLE II. Quartz on nickel with air in the chamber. 7.0 grams of quartz.

Date	Pressure	Electrometer reading 1	2	3	4	5	6	Av.	Volts
4/26/43	30	26.7	25.2	25.9	25.8	26.2		26.0	53.3
4/27/43	31	25.7	25.8	26.6	26.3	25.4	26.1	26.0	53.3
7/ 2/43	30	25.3	24.6	24.7	24.0	24.1		24.5	49.4
5/ 6/43	7×10^{-4}	36.6	35.1	36.5	36.1	35.2	36.2	36.0	104
5/ 6/43	264	36.7	35.9	36.8	36.3	36.1		36.4	107
4/26/43	42	25.4	24.7	24.9	25.1	24.6	25.0	24.9	50.5

Particles of nearly uniform size were segregated by differential screening. The average particle radius was 0.02 cm. The volume of an average particle was computed from the diameter by assuming that the particles were spherical. An approximate figure for the number of particles used was then computed (Table I).

The gases used were air, oxygen, nitrogen, and hydrogen. No attempt was made to purify the gases other than to remove water vapor, mercury vapor, oil particles, and dust particles.

EXPERIMENTAL PROCEDURE AND RESULTS

In order that data taken on different runs be comparable, it was necessary that the surfaces

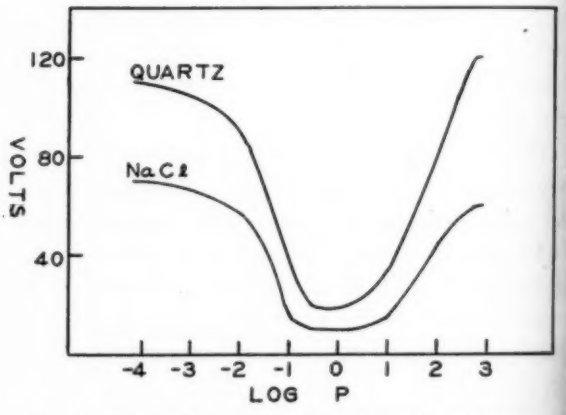

FIG. 2. Potential-pressure dependence for the entire range studied.

used be reproducible. I found that baking the apparatus for several days at 200°C in vacuum produced nickel, quartz, and NaCl surfaces with characteristics that were sufficiently reproducible for the needs of this experiment. The apparatus was given this treatment before each run.

For both quartz and NaCl a large number of measurements of potential were made at various pressures ranging from 7×10^{-5} to 760 mm of Hg. The rate of increase of pressure between measurements was slow so that the water vapor and dust traps could work efficiently. The rate of decrease of pressure between measurements was rapid.

At any single pressure from five to eight measurements of the potential were made from a similar number of inversions of the apparatus. In Table II are listed typical sets of measurements taken on different days and at different pressures. The difference between the highest and lowest reading of a set varies between 2 and 6 percent of the average of the set. The differences between average values taken for the same pressure on different days are of the same order of magnitude.

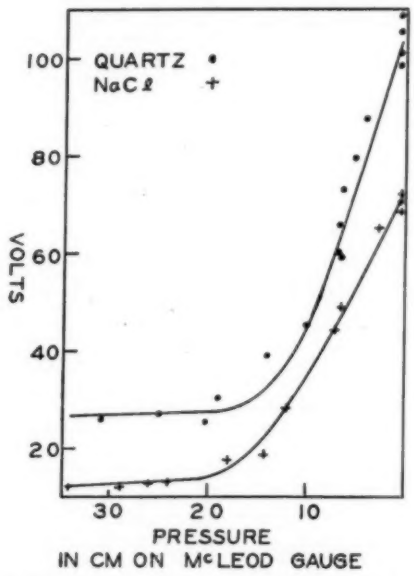

FIG. 3. Potential as a function of pressure—the low pressure range.

The sets of measurements were averaged and plotted (Figs. 2 to 6). In all, four sets of runs were made. The first run was made with 7.0 g of quartz particles in the apparatus. In the second run 8.1 g of NaCl were used and in the third run 4.7 g of quartz was used. The nickel flasks were then cleaned in dilute nitric acid and washed in distilled water. The fourth run was made with 7.8 g of freshly ground quartz.

In Fig. 2 potential is plotted against the logarithm of the pressure in order to give an over-all picture of the experimental results. The low

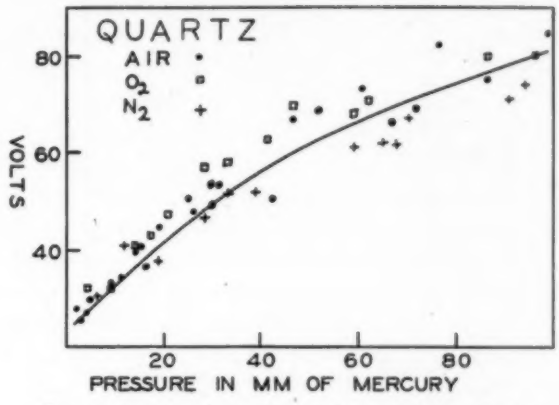

FIG. 4. Quartz: dependence of potential on gas in the chamber.

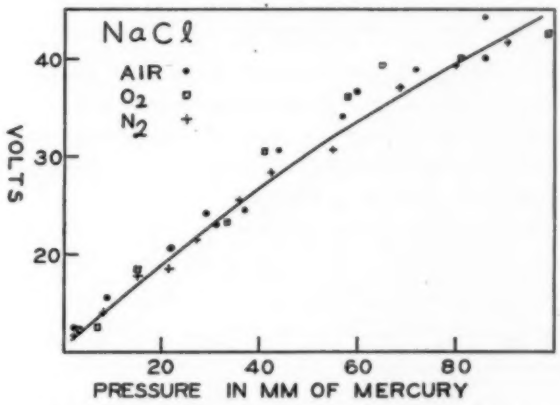

FIG. 5. NaCl: dependence of potential on gas in the chamber.

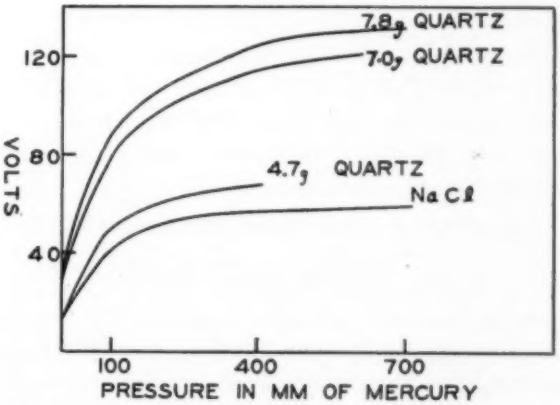

FIG. 6. Potential as a function of pressure—the high pressure range.

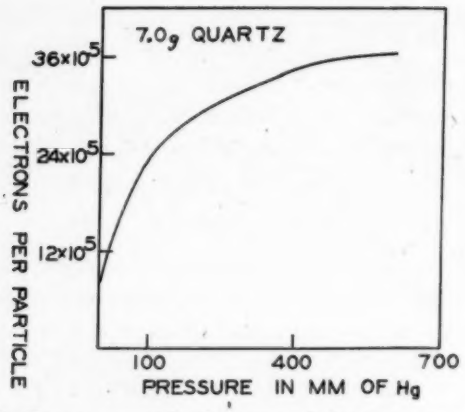

FIG. 7. Charge separation in terms of electrons per particle as a function of pressure.

pressure data are plotted in Fig. 3. For convenience the potential is plotted against McLeod gauge readings and thus against a quantity proportional to the square root of the pressure. In Figs. 4–6 the potential is plotted as a function of pressure in mm of Hg.

On the whole the quartz and NaCl behave quite similarly in the apparatus. But there was one very striking difference. After electrification the quartz particles were always negative while the NaCl particles were always positive.

From the capacity of the system and the measured potential, I computed the total charge separation, and the charge separation in terms of electrons per particle (Table III and Fig. 7). Although I have given the charge separation in terms of the electrons per particle, I do not wish to intimate that the mechanism of charging is that of a transfer of electrons as such. A possible mechanism will be discussed later.

DISCUSSION OF EXPERIMENTAL RESULTS

Air, Oxygen, and Nitrogen

On examining Figs. 4 and 5, we see that for the middle pressure range there is no significant difference in the behavior of oxygen, nitrogen, and their mixture air in this experiment. An examination of all of the data available shows that there is no real difference in their behavior at any pressure that I have studied.

The high pressure portion of the curve (Fig. 6) has a shape similar to that of the Langmuir adsorption isotherm.[8] The analytic expression for the adsorption isotherm is

$$x=aP/(1+aP), \tag{1}$$

where x represents the fraction of the surface covered by the adsorbed gas or rather the fraction of completion of an adsorbed layer. P represents the pressure of the gas being adsorbed and a is a constant depending on the gas and the surface. If the charge separation depends on the presence of adsorbed gas, we may thus postulate that the potential measured will be proportional to the fractional completion of the adsorbed layer. That is,

$$V=bx \tag{2}$$

or

$$V=baP/(1+aP). \tag{3}$$

TABLE III. Charge separation.

Non-metal	Pressure in mm of mercury	Volts	E.s.u.	Electrons per particle
Quartz	0.97	24.8	55.8	1.21×10^5
Quartz	47	67.2	151	3.28×10^5
Quartz	649	122	273	5.95×10^5
NaCl	0.81	12.1	27.2	0.72×10^5
NaCl	60	36.5	82.2	2.16×10^5
NaCl	701	60.5	136	3.59×10^5

On rearranging, Eq. (3) becomes

$$\frac{1}{V}=\frac{1}{abP}+\frac{1}{b}. \tag{4}$$

Then $1/V$ plotted against $1/P$ should be a straight line with slope $1/ab$ and intercept $1/b$. The reciprocal values of V and P are plotted in Figs. 8–11. The results approximate fairly well to

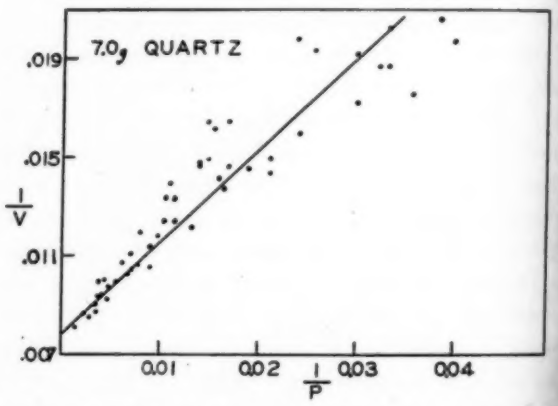

FIG. 8. 7.0 g quartz: plot of the reciprocal of the data for determination of adsorption isotherm constants.

[8] I. Langmuir, J. Am. Chem. Soc. 38, 2267 (1916); 40, 1361 (1918).

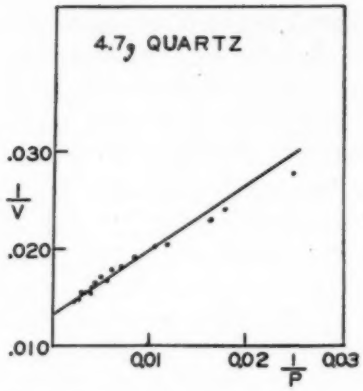

FIG. 9. 4.7 g quartz: plot of the reciprocal of the data for determination of adsorption isotherm constants.

a straight line. The constants obtained by fitting straight lines to the points are listed in Table IV.

These constants may be checked by comparing computed and observed values of the potential. Such a comparison is made in Tables V and VI. The data used for the comparison were selected at random from the available data. The differences between the observed and the computed values of the potential are of the same order of magnitude as the differences between individual values of the potential for a given pressure. We may safely say that Eq. (3) fits the experimental data within the uncertainty of the experiment.

Let us now inquire into the dependence of the constants of Eq. (3) on physical factors. The

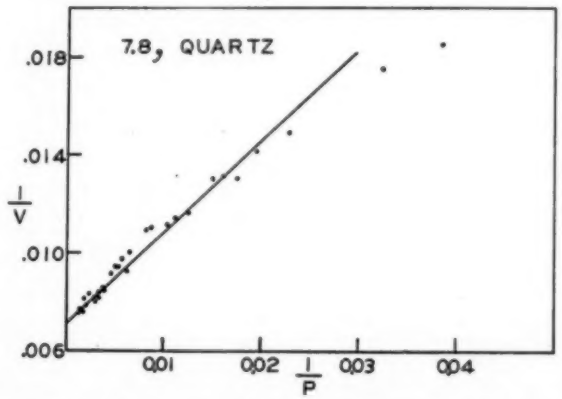

FIG. 10. 7.8 g quartz: plot of the reciprocal of the data for determination of adsorption isotherm constants.

magnitude of the constant a should depend on the nature of the gas adsorbed and on the nature of the adsorbing surface, and should be independent of the nature of the other solid surface involved. Since the magnitude of a did not change when NaCl was substituted for quartz (Table IV), it seems likely that the gas involved in the contact electrification had been adsorbed by the nickel. The other possible but less likely conclusion is that the adsorption behavior of quartz and NaCl toward the gases used is nearly identical.

The magnitude of the constant b should depend on the nature of both solids involved and on the number of particles that can make contact. We may assume that the charge separation is proportional to the area of contact and therefore to the number of similar particles that can make contact. In the last row of Table IV are listed the values for b adjusted to 10^8 particles. The values for the three quartz runs are of the same order of

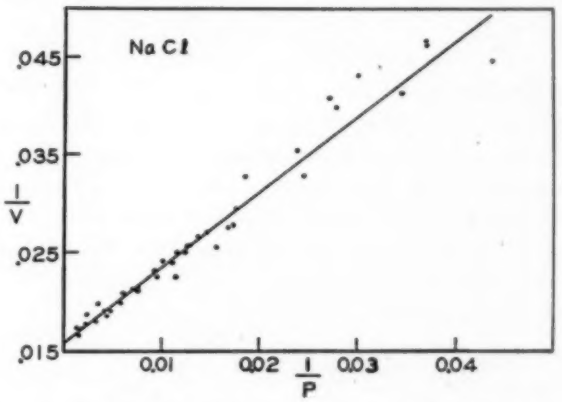

FIG. 11. NaCl: plot of the reciprocal of the data for determination of adsorption isotherm constants.

magnitude while the value for NaCl is quite different.

The action of NaCl is different from that of quartz not only in that the sign of the charge produced is different but also in the magnitude of the electrification per particle.

In arriving at Eq. (3), I assumed that the magnitude of the charge separation depended only on the portion of the surface covered with adsorbed gas. If the uncovered portion of the surface also makes a contribution to the charge separation, Eq. (3) must be changed to read

$$V=b\frac{aP}{1+ab}+c\left(1-\frac{dP}{1+dP}\right). \qquad (5)$$

If the uncovered portion of the surface does make such a contribution in this pressure range, it was so small as to have been obscured by the random variations in the results obtained.

TABLE IV. Adsorption isotherm constants—the high pressure range.

	Quartz 7.02 g	Quartz 4.74 g	Quartz 7.81 g	NaCl
Slope	0.375	0.64	0.37	0.80
Intercept	0.0078	0.0133	0.0070	0.0158
ab	2.67	1.56	2.70	1.25
b	128.0	76.0	143.0	63.3
a	0.021	0.021	0.019	0.020
Number of particles	1.41×10^{5}	0.95×10^{5}	1.57×10^{5}	1.16×10^{5}
b adjusted to 10^{5} particles	90.9	80.0	91.2	54.4

At pressures below 30 mm the data no longer fit Eq. (3), and at pressures below 0.1 mm the magnitude of the charge separation increases with decrease in pressure (Figs. 2 and 3). The potentials computed from Eq. (3) for pressures below 30 mm are lower than the observed potential. This difference is probably the contribution of the surface uncovered by the removal of the layer of adsorbed gas. The rapid increase in potential below 0.1 mm cannot be assigned to changes in the first layer of gas, for even at 1.0 mm but 2 percent of the surface is covered by this layer. This increase in potential must be caused by further changes in the surfaces making contact. Since the change in potential is still pressure dependent, it is reasonable to assume that the change involved is the removal of a second layer of adsorbed gas. Since the potential increases with decrease in pressure, we may assume that the surface uncovered by the removal of this second layer is making the major contribution to this potential. If this be the case, the dependence of potential on pressure should have the form of Eq. (6),

$$V=c/(1+dP). \qquad (6)$$

For pressures below 0.1 mm a decent fit of the data of Eq. (6) was obtained. The constants for the two runs with sufficient low pressure data to be so analyzed are listed in Table VII.

There are then at least two layers of adsorbed gas involved in the pressure range studied. At atmospheric pressure both layers are essentially complete and the charge transfer is large. As the pressure is decreased, one of these adsorbed layers is removed and the amount of charge transferred is reduced. The surface uncovered by the removal of the first layer of gas is then of lower activity than the same surface covered by the gas. The dependence of potential on pressure in this range is given by Eq. (3). At a pressure of 1 mm the first layer has been nearly removed but the second layer has not yet been touched. It is in this region that the charge transfer is a minimum. As the pressure is further reduced, the second layer of adsorbed gas is removed uncovering a surface of higher activity than that of the same surface covered by the adsorbed gas. The dependence of potential on pressure in this range is given by Eq. (6). It is probably a coincidence that the charge transfer obtained when both of the layers are complete is of the same order of magnitude as the charge transfer obtained when both layers have been removed.

We have not sufficient data to assign the second layer to one of the surfaces making contact. It may even be that what appears to be the removal of a single layer of adsorbed gas in each of the pressure ranges studied may be the nearly coincident removal of a layer of adsorbed gas from each of the surfaces.

Judging from the consistency of the results obtained with air, oxygen, and nitrogen, I believe that I have succeeded in reproducing the solid surfaces used by consistent treatment. I cannot, however, definitely state the exact nature of the surfaces used. It is probable that even at the lowest pressures used one or both of the surfaces were covered with a layer of adsorbed oxygen or nitrogen or both. The work of Roberts[9] indicates

TABLE V. Comparison of computed and experimental values of the potential. 7.0 g quartz.

P	$2.67\,P$	$1+0.021\,P$	Computed V	Observed V	V_0-V_c
4	10.7	1.084	9.9	27.2	+17.3
11	29.4	1.23	23.9	34.7	+10.8
25	66.8	1.53	43.6	50.6	+ 7.0
30	80.1	1.63	49.3	53.3	+ 4.0
42	112	1.88	59.4	50.5	− 8.9
61	163	2.28	71.5	73.0	+ 1.5
86	229	2.81	81.0	75.0	− 6.0
101	269	3.12	86.3	85.0	− 1.3
128	342	3.79	90.4	94.0	+ 3.6
147	393	4.09	96.5	96.0	− 0.5
208	555	5.37	103	103	0.0
251	670	6.27	106	101	− 5.0
375	993	8.81	112	118	+ 6.0
463	1235	10.71	115	116	+ 1.0
699	1731	14.61	118	122	+ 3.0

[9] J. K. Roberts, *Some Problems in Adsorption* (Cambridge University Press, 1939).

that at pressures below 10^{-5} a metal surface can be covered by more than one strongly held layer of adsorbed gas.

Hydrogen and Water Vapor

In an attempt to use hydrogen in the apparatus with quartz, I ran into many difficulties. I was unable to obtain any reproducible data in the hydrogen atmosphere. The charge collected was first of the same order of magnitude, then much lower than in the case of the other gases. I also had considerable difficulty in removing the hydrogen. Baking the apparatus at 200°C for three days while it was being evacuated by a mercury pump removed only some of the surface hydrogen. A run made in air after this treatment gave results that were much too low and which decreased with each inversion of the apparatus. It was only after the apparatus had been baked at 200°C and 20 mm of air for six days that reproducible results were again obtained.

The effect of hydrogen on the NaCl-nickel system was much the same as its effect on the quartz-nickel system.

The apparatus as it comes from the glass blower has been in contact with moist air. In this condition extremely low results are obtained for the charge separation. The charge separation can be returned to the higher reproducible values by baking out the apparatus at 200°C and a pressure of 7×10^{-5} mm of mercury.

Apparently water vapor and hydrogen act as poisons for the phenomena of contact electrification. The poisoning by hydrogen may have been a water vapor effect. It is probable that oxygen was never completely removed from the apparatus before the hydrogen runs. Then the processes involved in contact electrification may have catalyzed the chemical combination of hydrogen and oxygen to form a poisoning water layer on one of the surfaces. This mechanism would also account for the inconsistent results obtained with air when there was some residual hydrogen in the apparatus.

Mechanism of Contact Electrification

We do not have sufficient data to state definitely the mechanism of contact electrification of quartz or sodium chloride on nickel. The results indicate that the adsorbed gas molecules play an important part in the mechanism of charge transfer.

TABLE VI. Comparison of computed and experimental values of the potential. 8.1 g NaCl.

P	1.25 P	1+0.020 P	Computed V	Observed V	V_0-V_e
2	2.50	1.04	2.4	12.4	+10.0
5	6.25	1.10	5.7	14.1	+ 8.4
9	11.2	1.18	9.5	15.5	+ 6.0
22	27.5	1.44	19.1	20.5	+ 1.4
37	46.3	1.74	26.6	24.5	− 2.1
57	71.5	2.14	33.2	33.9	+ 0.7
72	90.0	2.42	37.1	37.5	+ 0.4
100	125	3.00	41.7	41.7	0.0
114	142.5	3.28	43.5	47.5	+ 4.0
131	164	3.62	45.4	44.7	− 0.7
172	215	4.44	48.5	49.2	+ 0.7
280	350	6.60	53.0	55.5	+ 2.5
457	572	10.14	56.2	57.0	+ 0.8
701	876	15.02	58.4	60.5	+ 2.1
760	950	16.20	58.6	58.1	− 0.5

TABLE VII. Adsorption isotherm constants—the low pressure range.

	7.0 grams quartz	8.1 grams NaCl
c	105	70
d	17	14

A substance like quartz does not accept or give up electrons easily, and therefore a mechanism involving the transfer of electrons is unlikely. But even amorphous quartz has holes in its short range order system from which ions of oxygen are missing. Some of these holes are on the surface. The presence of these holes should make it easier for negative ions to transfer from the nickel to the quartz. It is possible that the ions of the adsorbed gas molecules, oxygen or nitrogen, are transferred from the nickel to the quartz on contact and carry the charge.

In the case of sodium chloride this mechanism necessitates either the transfer of positive ions from the nickel to the sodium chloride or the transfer of negative ions from the sodium chloride to the nickel. The transfer of positive oxygen ions seems an unlikely process. An alternative process is the transfer of negative ions of gas molecules adsorbed by the sodium chloride or the transfer of chlorine ions from the surface of the sodium chloride. In either case the awkward point is the constancy of the constant a of Eq. (3) in all the experiments. If we accept this latter mechanism

and also the constancy of a, we must believe that nickel and sodium chloride behave in much the same way in adsorbing oxygen and nitrogen.

Further study of the phenomenon is needed before even an hypothesis for the mechanism can be made. In spite of the conflicts introduced by the ionic mechanism, it still seems more likely than an electronic mechanism.

ACKNOWLEDGMENTS

The author wishes to express his thanks and appreciation to Professor L. B. Loeb at whose suggestion the problem was undertaken and under whose direction it was carried out. In addition the author wishes to thank W. M. Taylor, Jr., and E. H. Guyon for their aid in the construction of the apparatus.

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The Mechanics of the Condensation of Gases

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An attempt is made to explain the phenomenon of condensation of the gas, on the basis of the classical mechanical model of a gas. The condensation of the gas is connected with a change in the topological structure of the energy surface in the $6N$-dimensional phase space. This occurs when the total energy is zero. The agreement with experiment is satisfactory.

1. INTRODUCTION

THIS paper is based on the assumption of a classical model for a gas: It is composed of N point masses attracting and repelling each other; the force between two molecules is derived from a potential $ar^{-n}-br^{-m}$, where r is the distance between them; a, b, n, m are positive constants (which we will consider as known from experiment) with $n>m$. We neglect internal degrees of freedom of the molecules. The differential equations governing the motions determine a family of trajectories in the $6N$-dimensional phase space. If the total energy of the system is fixed, equal to c, the trajectories are restricted to a $(6N-1)$-dimensional manifold $M(c)$. It is to be remembered that, since in a gas there is never any information as to the initial conditions of the individual molecules, this manifold (and the volume, see Section 5) is the only thing that is physically given.

The structure of $M(c)$ is analyzed for varying c and it is found that, for $c\geqq 0$, $M(c)$ has one type of structure, while for $c<0$ the structure is quite different. Roughly speaking, at $c=0$, $M(c)$ collapses from an infinite manifold ($c\geqq 0$) to a finite manifold ($c<0$). It would seem appropriate to identify the energy value zero (where the structure of the manifold changes) with a state of transition of the whole system. This transition would be of the nature of a phase transition, like condensation, for the gas. Accordingly we could say that a transition will take place when the total energy is zero. This last hypothesis combined with the usual statistical method leads to results which can only partially be checked by experiment. One has to make additional assumptions in order to obtain explicit expressions for the transition temperatures. These computations have been made for a number of gases, and a good agreement with experiment is found.

2. ANALYSIS OF $M(c)$

We consider a gas of N particles (x_i, y_i, z_i), $(i=1, 2, \cdots N)$ in xyz space. All masses are assumed to be equal to m. The distance between particle i and particle j is denoted by r_{ij}. With the mutual interaction between the molecules given by $f(r)=ar^{-n}-br^{-m}$ the total potential energy is

$$V=\sum_{i\neq j}\left(\frac{a}{r_{ij}^{n}}-\frac{b}{r_{ij}^{m}}\right). \tag{1}$$

The energy integral is given by

$$\frac{1}{2m}\sum_i (p_{x_i}^2+p_{y_i}^2+p_{z_i}^2)+V(x_1, \cdots z_N)=c. \quad (2)$$

Here $p_{x_i}=m(dx_i/dt)$, etc. Equation (2) defines the locus $M(c)$ in the $6N$-dimensional space of the variables $(x_1, \cdots z_N, p_{x_1}, \cdots p_{z_N})$.

Since the p's appear here only as the sum of their squares, it follows that the structure of the locus $M(c)$ is determined by the structure of the locus $V\leqq c$ in the space of $(x_1, \cdots z_N)$. The word structure is meant here in the sense of topology, i.e., of properties invariant under a continuous one-to-one transformation with continuous inverse.[1,2] However, we shall also remark on metrical properties.

To determine the structure of the locus $V\leqq c$ we first consider that of $V=c$, from which the former can be deduced. To that end we consider a ray from the origin in the $(x_1, \cdots z_N)$ space:

$$x_i=\lambda_i t, \quad y_i=\mu_i t, \quad z_i=\nu_i t, \qquad \sum_i (\lambda_i^2+\mu_i^2+\nu_i^2)=1. \quad (3)$$

This ray meets the hypersurface $V=c$ in points determined by the equation

$$(\alpha/t^n)-(\beta/t^m)=c, \quad (4)$$

where

$$\alpha=\sum a/\rho_{ij}^n>0, \quad \beta=\sum b/\rho_{ij}^m>0, \qquad \rho_{ij}=[(\lambda_i-\lambda_j)^2+(\mu_i-\mu_j)^2+(\nu_i-\nu_j)^2]^{\frac{1}{2}}. \quad (5)$$

It follows that for $c\geqq 0$ there is in general exactly one intersection point, while for $c<0$ there may be two points or one or none. There are certain exceptional rays which will never meet the hypersurface $V=c$, namely, those for which at least one $\rho_{ij}=0$. These rays correspond to collision configurations, configurations in which two or more molecules coincide. Because of the energy integral, such configurations can never arise for any finite value of c. The family of exceptional rays forms a point set of $3N-3$ dimensions and will hence have only a slight effect on the structure of the $3N$-dimensional locus $V\leqq c$.

[1] For references to topology, see P. Alexandroff and H. Hopf, *Topologie I* (Berlin, 1935), especially pp. 1–22.
[2] See W. Kaplan, Am. Math. Month. **59**, 316–323 (1942).

3. STRUCTURE OF $M(c)$ FOR $c\geqq 0$

In this case the above analysis shows that the locus $V\leqq c$ has the structure of the exterior of a $(3N-1)$-dimensional sphere (minus certain exceptional rays, corresponding again to collision). It follows that the energy integral defines a locus $M(c)$ which is unbounded in all directions with respect to the configuration coordinates $(x_1, \cdots z_N)$. This type of manifold may be realized as follows:

Start with the $(6N-1)$-dimensional sphere

$$\sum (p_{x_i}^2+p_{y_i}^2+p_{z_i}^2)+\sum (x_i^2+y_i^2+z_i^2)=1. \quad (6)$$

Now perform an inversion by reciprocal radii in the hypercylinder

$$\sum (x_i^2+y_i^2+z_i^2)=1.$$

That is, we replace each point $(x_1, \cdots z_N, p_{x_1}, \cdots p_{z_N})$ by the point $(x_1', \cdots z_N', p_{x_1}', \cdots p_{z_N}')$, where

$$p_{x_1}=p_{x_1}', \cdots p_{z_N}=p_{z_N}'; \qquad \frac{x_1}{x_1'}=\frac{x_2}{x_2'}=\cdots=\frac{z_N}{z_N'}; \qquad x_1x_1'\geqq 0, \cdots z_Nz_N'\geqq 0; \qquad (x_1^2+\cdots+z_N^2)(x_1'^2+\cdots+z_N'^2)=1. \quad (7)$$

Finally remove the points on the exceptional rays. The result is a locus having the same topological structure as $M(c)$ and having the same position[3] in the phase space as $M(c)$.

4. STRUCTURE OF $M(c)$ FOR $c\leqq 0$

Here we need a special lemma whose proof is omitted:

Lemma. *There is a constant $-\delta$, $\delta>0$, such that for $-\delta<c<0$ Eq. (4) has exactly two distinct roots on every non-exceptional ray. The least possible value of $-\delta$ is*

$$-\delta=\left(1-\frac{n}{m}\right)\left(\frac{m}{n}\right)^{n/(m-n)}\left(\frac{b^n}{a^m}\right)^{1/(n-m)}.$$

Suppose now $-\delta<c<0$. Then $V\leqq c$ holds between two hypersurfaces which are topologically equivalent to two concentric spheres. (Again exceptional rays must be removed.) Thus in this case the energy integral defines a locus which is essentially bounded. It becomes unbounded only

[3] See reference 1, p. 2.

in those directions which correspond to the exceptional rays. A further analysis shows that the structure of the manifold is the same as that of the topological product of a $3N$-dimensional sphere and a $(3N-1)$-dimensional sphere (minus certain exceptional points); that is, the same as the locus

$$\sum_{i=1}^{3N} u_i^2=1, \quad \sum_{i=1}^{3N+1} v_i^2=1$$

in the space of the variables $(u_1, \cdots u_{3N}, v_1, \cdots v_{3N+1})$. For $c \leqq -\delta$, $M(c)$ will collapse even further and shrink within itself until for $c=-\epsilon$, where $-\epsilon$ is a certain constant not yet determined, $-\epsilon > -\infty$, $M(c)$ disappears.

The value $-\delta$ is a second critical value for the manifold $M(c)$, but its physical significance has not been completely determined. Mayer and Mayer[4] in their statistical theory of condensation also obtain a second characteristic temperature. It is possible that our second change for $c=-\delta$ is related to their T_m.

5. $c=0$ AS A CRITICAL VALUE FOR THE ENERGY

The fundamental hypothesis introduced is that when the total energy of a gas changes from a small positive value to a small negative value a transition can occur. It is further assumed that this transition is the condensation of the gas. On the basis of the model assumed and these assumptions we can give the following description of the mechanism of condensation.

The molecules are pictured as mass points. Because of the nature of the potential energy function, a collision in the strict sense, i.e., some $r_{ij}=0$, is impossible. (Collisions in the sense of molecules coming so close together that they exert considerable forces upon each other will of course occur. In statistical mechanics one sometimes represents such a collision by a discontinuous collision between non-interacting elastic spheres, of finite radius; this is useful for computations, but does not represent the actual physical state of a collision.) This means that, if the walls of the containing vessel were not present, the motion of the gas particles would be completely without singularity; any motion would be determined by analytic functions for all time. The motion of the system would be represented by a smooth curve on the energy surface and it would be natural to correlate the properties of the system with the properties of this curve as a whole.

In order to take the walls into account, we notice first that collisions with the walls do not change the energy of the colliding molecules, and hence the total energy remains unchanged. The only effect of the walls is a discontinuous one: Every particle on reaching the wall changes its direction continuously but conserves its energy. Since the collisions with the walls are isolated events, between collisions the system will move with regularity as before. On the energy surface this means the following: The surface is cut off by a certain boundary (corresponding to a certain volume) so that the allowed portion is finite in extent. (Note that the velocity coordinates are always bounded to start with, owing to the energy equation.) A trajectory now appears as a curve which is smooth until a collision with the wall takes place. Thus the trajectory jumps from one point of the boundary of $M(c)$ to another point and starts from there another continuous path until again a collision with the walls takes place, and this process is repeated *ad infinitum*. Thus one trajectory appears as a multitude of pieces of different trajectories, no two pieces intersecting. The effect of the walls may accordingly be described as one of continuously varying the initial conditions.

Now the question is how the structure of the whole manifold $M(c)$ is related to the character of this multitude of trajectories and, in turn, how these are related to the structure of the gas. The explanation offered is as follows: If $c \geqq 0$ the surface is infinite in extent in a very strong sense, while when $c<0$ the infiniteness occurs only in a small set of directions. It follows, therefore, that when $c \geqq 0$ the probability of any orbit's leaving the finite part of the energy surface in a given time interval is large. Hence for most initial conditions the probability of meeting the walls in a given time interval is large. If $c<0$, the orbits will wander around for a long interval before reaching the boundary. In fact there is a very large probability that in a given time interval the

[4] J. E. Mayer and M. Goeppert-Mayer, *Statistical Mechanics* (John Wiley & Sons, Inc., New York, 1940), p. 308. J. E. Mayer and S. F. Harrison, J. Chem. Phys. 6, 87 (1938).

orbits remain completely in a bounded part of space, and never meet the boundary at all. Thus the discontinuous orbit is of a different nature, the system shows much less tendency to spread out, the gas condenses.[5]

It may be remarked that if the walls of the vessel are introduced as a force field, instead of in the qualitative way as above, the essential features of the argument are preserved. The case $c=0$ still appears as a critical value for the energy.

6. APPLICATION OF THE CRITERION

In order to test the validity of the assumptions made, a number of calculations have been made to compare the values obtained, on the basis of this theory, with the experimental values. In order to carry through the computation, one has to make some very strong additional assumptions, but it is reasonable to assume that at least the order of magnitude should be predicted correctly, if the proposed criterion has any value whatsoever.

We assume a gas of $N=M^3$ molecules in a cube of volume A^3 and we assume that the molecules form, as a first approximation (to be generalized below), a cubic lattice placed at the points $(h\alpha,\ k\alpha,\ l\alpha)$, where $h,\ k,\ l=0,\ \pm1,\ \cdots$ and $\alpha=A/M$ is the nearest distance between molecules. We use the potential function V as given above, but now use numerical values for the constants: $n=9$, $m=6$, a, b as given for various gases in Table I.

The total potential energy of this lattice can then be written as

$$V=\frac{N}{2}\left(\frac{aB}{\alpha^9}-\frac{bC}{\alpha^6}\right) \tag{8}$$

with

$$B=\sum\frac{1}{(h^2+k^2+l^2)^{9/2}},\quad C=\sum\frac{1}{(h^2+k^2+l^2)^3}. \tag{9}$$

Assume now that we have one mole of gas of volume v, so that $N=N_0$, Avogadro's number. Then, since $\alpha=A/M$, the potential energy can be written as

$$V=\frac{N_0^3}{2v^2}\left(\frac{aBN_0}{v}-bC\right). \tag{10}$$

The total kinetic energy we assume to be $3N_0kT/2$; this assumption defines the temperature T for this theory. The condition: kinetic energy + potential energy = 0 then becomes

$$\tfrac{3}{2}N_0kT=\frac{N_0^3}{2v^2}\left(bC-\frac{aBN_0}{v}\right) \tag{11}$$

or

$$T=\sigma b\left(\frac{1}{v}\right)^2-\zeta a\left(\frac{1}{v}\right)^3 \tag{12}$$

where

$$\sigma=\frac{N_0^2C}{3k},\quad \zeta=\frac{BN_0^3}{3k}. \tag{13}$$

If we determine the numerical values of the constants σ and ζ assuming a cubic lattice, we find

$$\sigma=7.5117\times10^{63},\quad \zeta=3.5939\times10^{97}. \tag{14}$$

This leads to predictions of the critical temperature T_c from the known values of the critical volume v_c. The results are listed in column three of Table I.

Now the assumption of a cubic lattice is not justified by experimental evidence. A better assumption would be that for all gases the spatial distribution is a unique one up to a similarity transformation. This is really equivalent to assuming the law of corresponding states, which, in a sense, is already implicit in this computation since we work with a two-parameter potential function. It would follow that a formula like (12) would still hold, with the same σ and ζ for all gases; however, their values are no longer given by (14). [This follows by the same reasoning that led to Eq. (4).] What the exact spatial structure is remains to be determined; the values of σ and ζ of course depend on that structure.[6] It is interesting to note that Lennard-Jones and Devonshire[7] in a series of papers on the theory of dense gases and liquids have worked with a

[5] A complete proof of the asserted properties of the orbits would require a delicate use of an ergodic-like hypothesis or one of metrical transitivity. See G. D. Birkhoff, Proc. Nat. Acad. Sci. **17**, 650–660 (1931), and J. C. Oxtoby and S. M. Ulam, Ann. Math. **42**, 874–920 (1941).

[6] In order to compute σ and ζ one must have some information about the distribution of the molecules in a dense gas. The actual computations would thus have to be along the lines indicated by Mayer and his co-workers. Cf. J. E. Mayer and E. Montroll, J. Chem. Phys. **9**, 2 (1941).

[7] J. E. Lennard-Jones and A. F. Devonshire, Proc. Roy. Soc. **A163**, 53–70 (1937); **165**, 1–11 (1938); **169**, 317–338 (1939); **170**, 464–484 (1939).

TABLE I. Compilation of results. Column 3 is based on a cubic lattice assumption. Columns 5 and 7 are based on the maximum point assumption.

	1 $a\cdot10^{23}$	2 $b\cdot10^{23}$	3 T_c pred.	4 T_c obs.	5 T_c pred.	6 v_c obs.	7 v_c pred.
Hydrogen	5.1	17.8	25	33.3	39	64.9	47.6
Helium	.51	2.37	4.4	5.3	9.3	57.8	35.8
Neon	3.5	14.5	43.6	44.5	45.1	41.7	40.0
Nitrogen	132	227	145	126	122	90.1	97.0
Argon	76.8	170	161	151	151	75.2	75.2
Krypton	193.6	346	219	211	200	107	93.5
Xenon	583	877	367	290	360	114	111

similar model and found it to give a consistent theory and good agreement with experiments.

In order to determine the best values of σ and ζ, we first note that according to our theory Eq. (12) should give the relationship between density and temperature of saturated vapor. According to (12) that temperature rises from zero to a maximum as the density increases, and then decreases. The first part of the curve is thus consistent with experiment; the second part has not as yet been explained. If the first part of the curve is correct, the maximum temperature should be T_c and the corresponding volume v_c. Using this we find

$$T_c=\frac{4}{27}\frac{\sigma^3b^3}{\zeta^2a^2},\quad v_c=\frac{3\zeta a}{2\sigma b}. \tag{15}$$

A simple way of determining σ and ζ would be to use the known values of T_c and v_c for one particular gas to compute σ and ζ by means of (15), and then use (15) further to compare the critical volumes and temperatures for other gases. The results are shown in columns 5 and 7 in Table I, with argon as the standard gas. The agreement so obtained is in general very satisfactory, especially when one considers the uncertainty in the force constants.

It should be remarked, however, that a very important factor contributing to this agreement is undoubtedly the fact that we use the law of corresponding states on the one hand, while we use some experimental data from the critical region to find σ and ζ on the other.[8] If we use the formula (15) for T_c and v_c directly, using for σ and ζ the cubic lattice values and for a and b values as given from experiment, we have a much more *a priori* theory. If this is done, values are obtained having the right order of magnitude; they are, however, not so close to the observed values as the ones computed in the other way. This is to be expected, since in this computation the cubic lattice assumptions enter explicitly. We can conclude that, in general, the agreement with experiment is satisfactory.

In a paper to follow a more detailed account will be given of the method. The connections with other theories of condensation will be given, and a more refined comparison with experiment will be undertaken.

[8] This point was first remarked to us by Professor G. E. Uhlenbeck.

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Interaction Between Longitudinal Current and Flux in a Nickel Bar

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It was found that reversing the current through the nickel alters the change of magnetism both when it is abruptly established and when it is abruptly stopped. In the sixteen different cycles considered, invariably a current flowing with the flux opposed magnetization, while a current flowing against the flux favored magnetization. A thorough analysis of the numerical data shows that in addition to the well-known "shock effect" equivalent to a mechanical jar, there is another effect which is reversed by reversing the direction of the current with reference to the flux. This effect apparently is not due to the circular flux set up by the current nor to any dissymmetry in the bar, but instead appears to be some kind of action between the current itself and the flux. This hypothesis was strengthened by a supplementary experiment showing that the resistance of a nickel bar is decreased when it is strongly magnetized by an alternating field.

INTRODUCTION

THE apparatus used in this investigation was identical with that used by Dr. Doolittle and myself in our experiments with residual magnetism in iron.[1] In our published article the various circuits and coils have been fully described[1] and only a brief recapitulation is given here. The sample to be tested was a short cylinder of nickel, kindly donated by the International Nickel Company. It was cored and slotted, so that its section is like a thick and nearly closed letter C. The purpose of the slot was to eliminate as far as practicable the circular flux set up by the longitudinal current through the bar. The sample was placed on the axis and at the center of a long solenoid in which a current of about two amperes sets up a strong magnetizing field. Surrounding the solenoid is a coil of many turns of fine wire connected to a ballistic galvanometer. The magnetizing current could be either made or broken abruptly by means of a switch, or gradually by moving sliders along the coils of three drum rheostats in series, thus varying the current from 0.0004 to 1.8 amperes in a few seconds.

The longitudinal current, to be here denoted simply by I, was always made or broken abruptly, and its effect, as Dr. Doolittle and I had previously observed, was equivalent to a mechanical shock causing a sudden increase or decrease in the bar's magnetism according to circumstances. This change in flux caused the galvanometer to deflect by an amount dependent on the violence of the mechanical blow or on the magnitude of the longitudinal current, and of course on the magnetic condition of the sample.

[1] H. A. Perkins and H. D. Doolittle, Phys. Rev. **60**, 811–817 (1941).

PROCEDURE

In the present investigation the value of I was always two amperes, and the magnetizing current was always brought, either quickly or slowly, to a final maximum very close to 1.8 amperes. The procedure involved carrying the nickel cylinder through a quasi-hysteresis cycle, and observing the deflections of the galvanometer at the vertical ends of the cycle as shown in Fig. 1. As these deflections are proportional to the change in flux caused by making or breaking I, they alone are quoted in what follows which is only concerned with relative values.

Sixteen cycles were examined, each of which may be represented qualitatively by one of the diagrams in Fig. 1. In (a), the nickel cylinder was magnetized following a curve similar to a typical $B-H$ curve from a to b, while the current I was flowing through it. At b, I was stopped. This caused an increase in magnetism bringing the flux abruptly to c which may be regarded as the normal condition for that field intensity. The field was then decreased to d when I was reestablished and the condition at a was recovered by a decrease of flux proportional to da. The increase from b to c and the decrease from d to a indicate a hysteretic lag which always occurs in

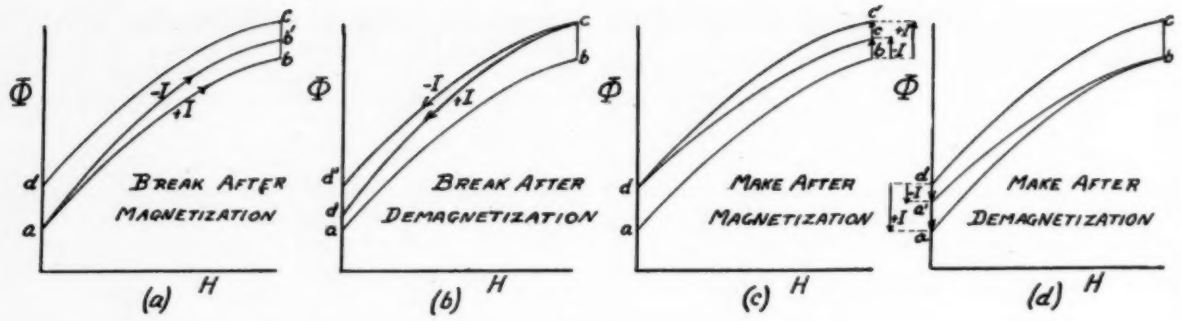

FIG. 1.

nickel, because the property of "overshooting the mark" previously found in Norway iron[1] does not exist with nickel or cast iron.

It should be noted that Figs. 1 are misleading in one respect. They exaggerate the deflection of the galvanometer at the two ends of the cycle. Although these "throws" were often over 100 scale divisions, the reading that would have been produced by the abrupt magnetization or demagnetization of the cylinder would have run to a thousand or more. An accurate drawing of such a cycle would show two curved lines very close together.

Another point to be noted is the absolute necessity of going through each cycle four or five times before any readings are made, in order to get the nickel accustomed to the regime. Nickel, and iron even more so, have amazing memories, and it takes considerable treatment to make them forget their immediate past. Taking this precaution, a given cycle in the case of nickel could be repeated again and again, or recovered later on with surprising consistency. Unfortunately Norway iron did not behave so well, although some of the same effects as with nickel were observable.

The sixteen cycles were obtained from the various possible combinations of the following factors. The magnetizing field may be established or discontinued either abruptly or slowly. The current may be flowing while the flux is increasing and not flowing while it is decreasing, or flowing during decreasing flux and not flowing during increasing flux. It may also flow during both processes or not flow during either except at the ends of the cycle where a momentary opening or closing of the switch is needed to obtain the galvanometer throw indicating the change of flux. As there are two such abrupt changes for each cycle, indicated by *bc* and *da* in Figs. 1, the 16 cycles give 32 different readings of the galvanometer. Then by reversing I, we obtain 32 more making a total of 64 distinct observations. Each of these observations was the average of four similar readings which never varied more than about seven percent from the average and generally very much less.

As a result of a careful study of the 64 different cases observed, it was found that, without exception, in stopping the longitudinal current I after magnetization, or in starting it after demagnetization, the galvanometer throw is greater when I is with the flux (denoted by $+I$) than when it is against it (denoted by $-I$). In starting I after magnetization or stopping it after demagnetization, the galvanometer throw is less when I is positive, and greater when it is negative. The meaning of these differences may be understood by referring to Figs. 1. In (a) the deflection due to the flux change $b'c$ when $-I$ flowed during magnetization is less than that due to bc when $+I$ was flowing. Thus $-I$ favored the growth of the flux while $+I$ held it back to the lower point b.

Figure 1(b) shows what happens at the other end of the cycle when I flowed during the process of demagnetization. When the current is with the flux ($+I$) the curve descends to d, while with $-I$ it only goes to d'. Thus there is more residual magnetism when the current is against the flux than when it is with it. So here again $-I$ favors the magnetic state as compared to $+I$ which presumably opposes it.

Figures 1(c) and (d) illustrate abruptly starting I at the two ends of the cycle. In (c) starting $-I$ causes the larger throw (b to c') indicating a

TABLE I. Results for eight different cases.

Procedure	Reading when I flows in nickel $+I$	Reading when I flows in nickel $-I$	Relative decrease of deflection when I is reversed	Reading when I flows in axial wire $+I$	Reading when I flows in axial wire $-I$	Relative decrease of deflection when I is reversed
1. Break after slow magnetization	+64	+50	22%	+66	+64	3%
2. Make after slow magnetization	+51	+58	12%	+59	+64	8%
3. Break after quick magnetization	+43	+34	21%	+54	+51	6%
4. Make after quick magnetization	+31	+40	23%	+49	+54	9%
5. Break after slow demagnetization	−100	−140	29%	−113	−120	6%
6. Make after slow demagnetization	−146	−105	28%	−120	−112	7%
7. Break after quick demagnetization	−23	−49	53%	−47	−45	4%
8. Make after quick demagnetization	−51	−25	51%	−48	−48	0%

greater increase of magnetism than that caused by starting $+I$ which carries the flux only to c. In (d) starting $-I$ after demagnetization brings the flux from d to a', while starting $+I$ brings it to a. But a' indicates more residual magnetism than a, so in this fourth case also, $-I$ favors the magnetic state while $+I$ tends to diminish it.

The average differences expressed in galvanometer scale divisions between the effects of $+I$ and $-I$ for the four cases just described and based on all 64 observations are as follows:

1. Break after magnetization: 12 mm due to bb', Fig. 1(a);
2. Break after demagnetization: 33 mm due to dd', Fig. 1(b);
3. Make after magnetization: 8 mm due to cc', Fig. 1(c);
4. Make after demagnetization: 34 mm due to aa', Fig. 1(d).

It is evident that the phenomenon is much more pronounced after demagnetization than after magnetization. This is to be expected, because as we approach saturation the magnetic state becomes increasingly rigid. Also the relative changes of flux are found to be larger after quick than after slow demagnetization, although the actual changes involved are smaller, because quick demagnetization results in a smaller residual flux than when the magnetism is gradually reduced.

In Table I are listed eight essentially different cases. Each of the galvanometer throws indicated under $+I$ or $-I$ is the average obtained from four different cycles. These four values agree fairly well among themselves, especially after demagnetization, which shows that an individual case is not much influenced by the rest of the cycle.

DISCUSSION OF RESULTS

It will be seen that after magnetization the deflection is listed as positive to indicate an increase of flux, while the negative sign after demagnetization indicates a decrease as would be expected. The first two columns of figures are those already discussed. The other values for an axial current will be explained later. The column indicating the relative change due to a reversal of I shows that in some cases the change is very marked and is never less than 12 percent. This effect can be explained by supposing two major effects, A and B, and a third minor one, C. The first of these, A, is the well-known "shock effect" which alters the magnetization as a result of a mechanical jar or a sudden rush of current, independent of its direction. The second, "B effect," accounts for the change of deflection when I is reversed. It is this phenomenon with which the present discussion is chiefly concerned, and it is apparently not accounted for by classical electromagnetic theory.

The C effect is due to the action of the circular flux set up in the bar by the current flowing through it. This circular flux might be supposed to explain the effect of reversing I, but it was shown to be much too small to account for the magnitude of the observed phenomena. A crucial test proving this fact was made as follows. Instead of sending I through the nickel, it was carried in a wire along the axis of the cored cylinder. Thus only two effects were possible, namely, shock and the interaction between the circular flux set up by the axial current and the longitudinal flux in the bar. This interaction, in spite of the slot, designed to minimize circular flux, must also occur to some extent when the

current flows through the metal. But calculation, as stated in the earlier article,[1] shows that the circular flux is 2.5 times as large when the current is axial, so we should now expect it to assume some importance. Thus though the shock effect must be independent of the direction of I, the influence of the circular flux should result in somewhat different galvanometer throws when I is reversed.

The results obtained with the axial current, keeping all other conditions the same as before, are given in the last three columns of Table I. It will be seen that the galvanometer throws are similar to those previously obtained. But, though of the same general magnitude, the lower value of the various pairs is now nearly equal to the higher values and the proportional decrease does not exceed 9 percent obtained in case number 4 as compared to 23 percent previously observed. It goes down practically to 0 percent as compared to 51 percent in case number 8. The differences caused by reversing I when it is axial are probably due to circular flux, but though observable in all but case number 8, it is relatively small, and since the axial current is 2.5 times as effective in setting up a circular flux as the current through the bar itself, we may be justified in dividing the percentages by that figure. Then the largest proportional change of 9 percent would be reduced to 3.6 percent as compared to 23 percent when the B effect as well as the C effect is present. Thus the C effect is shown to contribute something but not a great deal to the change of deflection brought about by reversing I.

ANALYSIS OF RESULTS

Since we are now justified in neglecting the relatively small C effect as an explanation of the observed phenomena, there remain the A and B effects to be considered. These may be separated as follows. The shock effect in nickel always involves an increased flux after magnetization and a decreased flux after demagnetization, for nickel, unlike Norway iron, shows no evidence of "overshooting the mark." This means that shock tends to carry the flux nearer to its final stable value. But the B effect tends to help or oppose this return to a stable state according to its direction, and whether or not reaching the stable state calls for an increase or decrease of magnetization. Thus if we call the change of flux caused by shock ΔA, and that caused by the B effect ΔB, we have $\Delta A+\Delta B=M$ when the effects are in the same sense, and $\Delta A-\Delta B=N$ when they are opposed. Thus $\Delta A=\frac{1}{2}(M+N)$ and $\Delta B=\frac{1}{2}(M-N)$. Applying this method to case number 7 (the most conspicuous one) we obtain from the galvanometer readings (proportional to ΔA and ΔB) $\Delta A=\frac{1}{2}(23+49)=36$ which is proportional to the shock effect, and $\Delta B=\frac{1}{2}(49-23)=13$ which represents the B effect caused by $+I$ flowing during the process of demagnetization. The same analysis applied to case number 3, for instance, gives $\Delta A=38.5$ and $\Delta B=4.5$. If this analysis is applied to the corresponding cases when the current is axial, we may separate the A and C effects. Thus case number 7 gives $\Delta A=46$ and $\Delta C=1$, while in case number 3 $\Delta A=52.5$ and $\Delta C=1.5$. These results seem to justify ignoring the effect of circular flux in estimating the B effect when the current flows through the bar, although strictly speaking the values 13 and 4.5 obtained above are partly due to C as well as B but with ΔC considerably smaller than 1 and 1.5 for reasons already mentioned. Aside from circular flux, other possible explanations of ΔB caused by reversing I are dissymmetry of the bar itself due to crystalline structure, and the effect of the magnetic field of the earth to which it was parallel. Both these were shown to be negligible by reversing the magnetizing current. When this is done, substantially the same values as before are recovered provided I is also reversed. Several cycles were carried through in this way, with practically the same galvanometer readings as before.

DISCUSSION OF RESULTS

To sum up the results, it seems safe to conclude that the B effect represents some apparently new kind of interaction between current and flux Φ, favoring Φ when it flows in the opposite direction and opposing Φ when in the same direction. Thus we may imagine a pure shock effect caused as by a mechanical jar tending toward magnetic stability with a bias due to B added to it or subtracted from it according to the direction of the current with respect to the flux. This effect exists in nickel, but to a much smaller degree, if at all, in a bar of Norway iron which was carefully ex-

amined with a view to detecting a possible alteration in the galvanometer deflection when I is reversed.

Finally the question naturally arises: Is there a reciprocal phenomenon? That is, if making or breaking a longitudinal current, aside from the effect of shock, alters the magnetic state of the iron, does a varying magnetic field alter the current? This would mean a change in the effective resistance of the bar. To test this possibility, bars of soft iron and of very pure nickel kindly donated by the International Nickel Company were wound with a double-layer solenoid and magnetized by an alternating current of 60 cycles. Their resistances both magnetized and demagnetized were measured in a Kelvin double bridge. The iron exhibited no observable change of resistance, but the resistance of the nickel was decreased by more than 0.6 percent when the field was applied, and remained at the lower value as long as the alternating current was flowing. There was of course a decided gradual increase of resistance caused by rising temperature, but during the short time needed to measure its new and smaller value, when the alternating field was applied, the rise due to higher temperature was much too small to obscure the phenomenon.

It has long been known that a steady longitudinal field raises the resistance of nickel measured in the same direction, that the relative increase, $\Delta R/R$, is unaltered by reversing the field, and that this magnetoresistance effect is associated with the still older phenomenon of magnetostriction.[2] But apparently the effect of a rapidly alternating field has not been examined, and it is certainly surprising to find that such a field *lowers* the resistance even more than it is increased by a steady field. This increase was also measured with similar excitation by the author and it was found to be between 0.5 and 0.6 percent instead of over 0.6 percent with an alternating current. In both cases the flux density was about 1900 gauss which was unnecessarily high, as the saturation flux density for this effect is of the order of 100 gauss.

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[2] L. W. McKeehan, Phys. Rev. **36**, 948 (1930).

Experiments on Electrets

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(Received March 17, 1944)

WE have performed a systematic study of the electret effect,[1] guided by the idea that it must be related to the various other aspects of anomalous behavior, of solid dielectrics. The manufacture of electrets, as it is usually accomplished, involves a heat treatment. Prior, therefore, to the investigation of the fields of electrets, we studied the influence of the temperature on the dielectric absorption. A preliminary report of our results is presented here.

TERMINOLOGY

Following Gemant,[1] we call homocharges the true (surface or space) charges, the sign of which is the same as that of the corresponding polarizing electrode, and we call heterocharges the true charges of contrary sign. Both kinds of charges coexist in a polarized dielectric. In the shorted capacitor, their superposed fields induce on the electrode a charge q, which is measured by lifting the electrode. An apparent surface charge $-q$ is then ascribed to the dielectric.

INFLUENCE OF TEMPERATURE ON DIELECTRIC ABSORPTION

Here we are not concerned with the measurement of isothermic current—time curves, but with non-isothermic effects, which have a bearing on the electret effect. Their existence, suggested by former papers,[2] could be verified: A capacitor containing an absorptive dielectric (like carnauba wax) is charged at a high temperature and then, always connected with the voltage source, cooled until a considerably lower temperature is reached; finally it is short-circuited. Nevertheless, only a partial discharge occurs. A part of the quantity of electricity stored up during the preceding charging cycle behaves like a "frozen" charge and is retained within the dielectric as long as the temperature is kept low. The value of the frozen charge depends on the nature of the dielectric, on the charging time, and on the difference of temperature between charge and discharge, but it is independent of the rate of cooling. The explanation of the effect is found in the enormous increase, with rising temperature, of both the charging and the discharging velocity. The true absorption capacitance of capacitors with solid dielectrics is frequently much higher than is usually believed. But the charging velocity at room temperature is so low that at such temperatures the capacitors are never charged completely. On the contrary, in the first stage of our experiment the capacitor is almost completely charged in a brief time, because it is charged at a high speed at the high temperature. In the second stage (beginning with the short circuit), the capacitor is far from being completely discharged even after a long time, because at the low temperature it is discharged at a low speed; thus the dissipation of the absorbed quantity of electricity is so small that a part of it is trapped in the dielectric. Whatever may be the nature of these frozen charges, they must produce electric fields. This type of dielectric absorption yields therefore necessarily the electret effect; but it does not explain everything, because (a) the mechanism, which according to current theories (dipole and ionic theories) is responsible for dielectric absorption, accounts only for the heterocharges, not for the homocharges of electrets; and (b) electrets can be made without heat treatment.

DECOMPOSITION OF THE CURRENT INTO DISPLACEMENT CURRENT AND CONDUCTION CURRENT

We have investigated the nature of the homocharges by varying the intensity and time of application of the polarizing field, by modi-

[1] Cf. M. Eguchi, Phil. Mag. **49**, 178 (1925). S. Mikola, Zeits. f. Physik **32**, 476 (1925). E. P. Adams, J. Frank. Inst. **204**, 469 (1927). M. Ewing, Phys. Rev. **36**, 378 (1930). R. D. Bennett, Phys. Rev. **37**, 103 (1931). A. Gemant, Phil. Mag. **20**, 929 (1935). G. Groetzinger and H. Kretsch, Zeits. f. Physik **103**, 337 (1936). P. A. Thiessen, A. Winkel, and K. Herrmann, Physik. Zeits. **37**, 511 (1936). W. M. Good and J. D. Stranathan, Phys. Rev. **56**, 810 (1939). B. Gross, Anais Acad. Bras. Ci. **15**, 63 (1943).

[2] H. Frei and G. Groetzinger, Physik. Zeits. **37**, 720 (1936). O. G. v. Altheim, Ann. d. Physik **35**, 417 (1939).

fying the temperature, and by speeding up the disappearance of the field of the electret by heating. During the discharge, the total current J and the charge q of one of the electrodes were measured. According to the equation $J=i+dq/dt$, this method permits us to split up the current into its two components, the conduction current i and the displacement current dq/dt, both referred to the interface dielectric-electrode. The conduction current originated by the decay of a homocharge has the usual direction of the discharging current, but the corresponding displacement current flows in the direction of the charging current; just opposite in direction are the currents for heterocharges. The decay of the homocharges proceeds partially by conduction across the interface dielectric-electrodes, partially by conduction *within* the dielectric. If the second effect is sufficiently strong, the total discharging current may reverse.[3] These current reversals are not to be confused with those which are accounted for by the principle of superposition and which are observed after a brief application of a voltage of one sign succeeding a prolonged application of a voltage of the same value but contrary sign.[4]

MEASUREMENTS ON CURRENTS AND CHARGES IN ELECTRETS

Even without heat treatment, numerous dielectrics produce strong fields after the application of high voltages (cf. S. Mikola).[1] An important factor is the duration of the polarization. The results are particularly clear for carnauba wax. (1) If the time of polarization has been very short, the apparent charge—with the sign of a homocharge—decays at first rapidly. The decay is associated with a strong displacement current so that the total current flows in the same direction as during the charging period. Later the apparent charge becomes more stable and the current goes down. When the temperature is increased, the charge disappears completely and a transient displacement current is raised. (2) If the time of polarization has been long, shortly after the short circuit an increase, and later a maximum of the apparent charge (which again has the sign of a homocharge), are observed. The current is now mainly conduction current. It is the usual discharging current familiar from the numerous experiments on anomalous currents in solid dielectrics. A later increase of the temperature is followed by an immediate decay of the apparent charge, accompanied, as in the first case, by a displacement current.

The usual heat treatment produces the same effect as an enormous prolongment of the duration of the polarization. The value of the temperature itself has only quantitative influence. It need not be raised above the melting point of the substance. Strong permanent electrets can be obtained by polarization in the solid state. It becomes now evident that immediately after the short circuit the sign of the apparent charge is always that of a heterocharge; but sooner or later a change of polarity occurs and it turns into that of a homocharge. Simultaneously a strong discharging current, almost entirely conduction current, is observed. If the temperature of the shorted system is raised temporarily, the value of the apparent charge (homocharge) is greatly enhanced, but it returns slowly to its former level after room temperature has been reached again. If the temperature is raised and maintained high for a long time, the apparent charge disappears after a transitory increase and a strong discharging current surges, again a conduction current.

The systematic deviations from the principle of superposition, which at high voltages become obvious, are accounted for by the presence of homocharges.

Ionization in an air gap eventually existing between dielectric and electrode is not significant. We have observed current reversals indicating the existence of homocharges on hard rubber disks with silvered surfaces.

THEORY OF ELECTRET BEHAVIOR

Two mechanisms act simultaneously: (1) dielectric absorption related to the movement of ions or the orientation of dipoles in the interior of the dielectric originates heterocharges; (2) conduction in the interphase dielectric-electrode which originates homocharges. Without heat

[3] Reversals of a similar kind seem to have been observed by H. H. Race, Trans. A. I. E. E. **47** (October, 1928) and by Sadakichi Shimizu, Tohoku Univ., Sci. Reports **21**, 29 (1932), see especially p. 48.

[4] Cf. V. Karapetoff, Trans. A. I. E. E. **45**, 124 (1926).

treatment, the heterocharges, with a smaller relaxation time, disappear spontaneously after the system has been shorted, and a pure homoharge remains. The velocity of the transition depends on the degree of dielectric absorption, which in its turn is determined by the duration of the polarization. After a heat treatment, a considerable fraction of the heterocharge is preserved besides the homocharge in the shorted system. The resulting field depends on the values and spacial distribution of both charges; its transitory increase during a later heating is another consequence of the smaller relaxation time of the heterocharges.

In polar substances, the formation of heterocharges is due mainly to dipole orientation. Proportionally to the value of these charges the electric field increases in the interphase dielectric-electrode. With sufficiently high field strengths, conduction currents surge in the interphase and, consequently, ions (or electrons) are fed into the dielectric or extracted from its surface and transferred to the electrodes. Accordingly, homocharges appear, initially in the form of surface charges, later spraying over a certain depth within the dielectric.[5] This process, resulting in a weakening of the field, reduces the conduction currents. With the short circuit, a part of the dielectric polarization, in phase with the electric field, disappears immediately; the rest follows more or less slowly. In consequence of this decay of the heterocharge, the homocharge component begins to prevail in the resulting field. Again conduction currents play a role, now carrying away part of the homocharge. It must be kept in mind that there is no recombination between the two types of charges. Thus, the homocharge is released in the same measure as the heterocharge decays. If a part of the heterocharge is frozen in, the relaxation time for its decay is so long that the homocharge in its turn is conserved over extremely long periods of time. Such an electret possesses really a volume polarization like a magnetized iron bar, but the field becomes disturbed by conduction effects non-existing in magnetism. If an electret with strong homocharges is cut into two pieces of about equal dimensions, both surfaces of the same piece should show charges of the same sign. But conduction currents surge at the newly created surface in contact with metal, the internal field of the electret now playing the same role as previously did the external field. So a change of polarity takes place and each piece becomes a dipole.

The effect, which manifests itself in the appearance of homocharges, must interfere in many problems of dielectric behavior, like increase of dielectric loss at high voltages and rupture. Its closer investigation may reveal further interesting features.

[5] This picture is very similar to that given by M. Faraday, *Experimental Researches in Electricity, Volume I* (Richard and John Edward Taylor, London, 1839), who has already known the existence of homocharges: "The effects appear to be due to an actual penetration of the charge to some distance within the electric, at each of its two surfaces, by what we call *conduction;* so that, to use the ordinary phrase, the electric forces, sustaining the induction, are not upon the metallic surfaces only, but upon and within the dielectric also, extending to a smaller or greater depth from the metal linings." (Art. 1245, p. 390.)

ACKNOWLEDGMENTS

I wish to express my gratitude to Professor E. L. da Fonseca Costa who made this study possible. I am indebted to Mrs. L. Ferreira Denard for the execution of all experimental work on dielectric absorption and to Dr. J. Costa Ribeiro for many stimulating discussions.

Letters to the Editor

PROMPT publication of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is the third of the month. Because of the late closing date for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not in general exceed 600 words in length.

On a Recent Paper by Jaffé

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June 22, 1944

I HAVE been interested in comparing a paper, "On the currents carried by electrons of uniform velocity," by George Jaffé,[1] with an earlier paper on the subject by C. E. Fay, A. L. Samuel, and W. Shockley.[2] These treatments agree in all cases in which the current is not "space-charge limited." The difference may be illustrated by the following special case.

Suppose we have a thermionic cathode, a positive grid close to it, and an anode at the same potential as the grid, spaced many times as far from the grid as the grid is from the cathode. Imagine that we gradually increase the temperature of the cathode, and hence the current from it, until the cathode current finally becomes space-charge limited.

According to Fay, Samuel, and Shockley, the entire cathode current can be transmitted through the grid-anode region until a "limiting current" is reached. For cathode currents greater than this, and also as an alternative possibility for currents somewhat smaller, they would regard as valid an expression which involves a potential minimum at cathode potential between grid and anode, with part of the electron flow turned back toward the cathode at this potential minimum. Under this condition the anode current is considerably less than the "limiting current." Clearly, Fay, Samuel, and Shockley regard this condition only as representing approximately the actual solution with thermal velocities, for they say, "The assumption of equal energy electrons leads to indeterminacy at planes where the potential and its gradient are both zero. At these planes the existence of non-uniform electron velocities will be recognized insofar as it provides a selective mechanism to resolve the mathematical indeterminacy."

On the other hand, Jaffé rejects completely any expressions implying zero electron velocity and infinite charge density at a plane, and according to his treatment the final value of the anode current is the "limiting current" of Fay, Samuel, and Shockley, mentioned above.

Jaffé says, "Now, the velocity of the electrons naturally becomes zero at a point where $\eta=-1$ [see (3)], and therefore the density n would have to be infinite. We are of the opinion that no solution should be admitted which makes a physically measurable quantity infinite. We, therefore, conclude that the limiting case $X_1=0$ (on branch II) should be excluded even as a possible form of unstable discharge." This, of course, excludes all solutions for which a part of the current is turned back, and indeed, Jaffé says in his introduction, "In general, the minimum will not be low enough to stop electrons which have entered the discharge space. In spite of this fact the current will be space-charge limited (for sufficiently low values of the potential) because the solution of the problem does not exist for any given value of the plate distance unless the current is below a critical value. Thus the space-charge limitation is caused, in this case, not by the fact that some electrons return to the cathode, but by the fact that only a limited number of them can get into the discharge space."

It is pretty clear experimentally that electrons can be turned back in a space between approximately plane electrodes. The writer believes that expressions given by Fay, Samuel, and Shockley which involve a plane of zero velocity and infinite charge density (the charge is not infinite) are physically acceptable as representing a limit which the correct solutions involving thermal velocities must approach as the parameter expressing the ratio of the thermal energy to the energy at the entrance or exit plane is made very small.

In regard to stability, Jaffé's argument that in cases in which the velocity does not go to zero, the solution with the lower potential minimum (Fay, Samuel, and Shockley C overlap solution) is unstable because "it would require work to change the discharge with the higher potential curve to that with the lower potential curve" is not decisive, as by this criterion a bottle is unstable when standing upright. Fay, Samuel, and Shockley also regard the C overlap solution as unstable, and there are reasons to suspect that it is completely unstable.

In a paper, "Vacuum-tube networks," by F. B. Llewellyn and L. C. Peterson,[3] the a.c. problem is treated. For an undisturbed current injected into a diode, the relation between a.c. (or perturbation) voltage V across the diode and a.c. (or perturbation) current I in the diode is given by an expression which for very low frequencies reduces to

$$V=I(u_a+u_b)(T/\epsilon)(1-\zeta+\zeta\beta/3)/\beta.$$

Here u_a and u_b are velocities at the entrance and exit planes, ϵ is the dielectric constant of vacuum, T is transit time across the diode, ζ is a "space-charge factor" which for the C overlap solution is always greater than unity, and $\beta=i\omega T$. We note that for certain small values of β such that

$$\beta=3(\zeta-1)/\zeta,$$

current oscillations can occur in a short-circuited diode ($V=0$). We also note that for ζ a little less than unity the oscillation starts as an exponential decay of perturbation current with time, while for ζ a little larger than unity the oscillation starts as an exponential increase of perturbation current with time, indicating an instability.

[1] George Jaffé, Phys. Rev. **65**, 91–98 (1944).
[2] C. E. Fay, A. L. Samuel, and W. Shockley, Bell Sys. Tech. J. **17**, 49–79 (1938). The same matter was treated by B. Salzberg and A. V. Haeff in the RCA Review (Jan. 1938).
[3] F. B. Llewellyn and L. C. Peterson, Proc. I.R.E. **32**, 144–166 (1944).

On the Currents Carried by Electrons of Uniform Initial Velocity

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July 3, 1944

IN a Letter to the Editor, Mr. J. R. Pierce has compared my recent paper under the above title[1] with an earlier treatment of the same subject by C. E. Fay, A. L. Samuel, and W. Shockley.[2] Though Mr. Pierce's representation of my point of view is entirely correct, I should like to add a few remarks since it seems to me that a point of principle is involved.

The paper of the three authors was not known to me until Mr. Pierce drew my attention to it. However, the treatment therein is based on the treatment of Langmuir and Compton[3] to which I have referred.

The original derivation of the Child-Langmuir formula was subject to the following two criticisms:

(a) It does not contain the slightest indication of the phenomenon of saturation.

(b) It makes the solution singular in a mathematical sense by admitting an infinite value of the space charge density (at the cathode).

Admittedly the treatment of L. and C. and of the later authors, in the case of constant initial velocity, is open to objection (b) as regards the plane where the potential minimum occurs. It was the only aim in my investigation to do away with the mentioned objections. By introducing a modified form of boundary condition, I obtained solutions which admit of no indeterminacy, which are regular everywhere (also in the case of space-charge limited currents), and which automatically change into the saturated solution when a critical potential is reached.

If Mr. Pierce and others are willing to admit infinite values of the space-charge density, there is nothing to be said against the treatment of L. and C. except that a very puzzling ambiguity arises. Which solution is to be accepted as the valid one, the regular or the singular one? From general physical and mathematical principles there can hardly be a doubt as to the answer. However, there is a further argument in favor of the regular solution.

When the space-charge limitation sets in, the minimum of the potential is not yet as low as its lowest possible value. (Thus in the case $\xi_0=10$ represented by Fig. 3 of my paper, the limitation sets in for $\eta_{\min}=-0.45$, and for real cases of thermionic emission the minimum would be still less low.) If now the limitation sets in (e.g., by an increase in the potential) according to the solution of L. and C., the minimum would have to jump suddenly to $\eta_{\min}=-1$. It seems to me that such a behavior is quite inadmissible and that, therefore, the regular solution has to be continued into the domain of space-charge limitation. This is exactly what my modified boundary condition achieves.

How nearly the singular solution approximates the regular one remains to be discussed. It is easy to show that the treatment of L. and C. approximates the regular solution in two limiting cases, namely, either (in the notation of my paper) when $E_0/E\to 0$, or when $\xi_0\to 5/3$, which means when $16\pi i/(v_0E_0)^{\frac{1}{2}}x\to 5/3$. In the former case the minimum will be slight ($\eta_{\min}\to 0$) and near the cathode; in the latter case it will be near the lowest possible value ($\eta_{\min}\to -1$) and near the anode.

In all other cases deviations must be anticipated which concern the potential distribution as well as the characteristic. It will be sufficient to show the discrepancy in the latter case. Particularly large deviations are likely to occur for relatively large initial velocities, i.e., in the limiting case $E_0/E\gg 1$. In this limit my formula (30) yields the value $j=8E_0^{\frac{1}{2}}$. On the other hand, it can be deduced from L. and C.'s formulae (293) to (295) that $j=E_0^{\frac{1}{2}}((1+\gamma)/\gamma)^2$, where $\gamma=(2i_s/i-1)^{\frac{1}{2}}$. Since the saturation value i_s is always larger than i, γ will be a number larger than unity, and the factor multiplying $E_0^{\frac{1}{2}}$ will be between $\frac{1}{2}$ and $\frac{1}{4}$ of the value in my formula.

I should like to stress once more that my treatment regards exclusively the idealization that all electrons have the same velocity normal to the cathode. It is only in this case that the electrons cannot build up a minimum sufficiently low to make them revert (unless a singularity is admitted). If there is a velocity distribution, it has been shown long ago, and I have stressed the fact in my paper that the slower electrons will revert in the field created by the swifter ones.

As to the question of stability, I agree with Mr. Pierce that it cannot be answered completely without a special investigation. Therefore, I should have called the solution with the lower minimum not the "unstable solution" but the "less stable solution." As a matter of fact, I did admit a certain stability for the "unstable solution"—otherwise I would not have drawn diagrams giving the potential distribution in such cases. For the main point under discussion this is quite irrelevant. I pointed out that the solutions which make the minimum as low as $\eta_{\min}=-1$ are on the unstable branch, but I excluded them as physically inadmissible because they make the space-charge density infinite.

[1] George Jaffé, Phys. Rev. 65, 91 (1944).
[2] C. E. Fay, A. L. Samuel, and W. Shockley, Bell Sys. Tech. J. 17, 49 (1938).
[3] I. Langmuir and K. T. Compton, Rev. Mod. Phys. 3, 191 (1931), see p. 239.

Retraction of Stressed Rubber

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July 3, 1944

THE process of the retraction of stressed rubber has been studied by us for some time, using several experimental methods. The most promising method employed was the use of a smoked revolving drum driven by a synchronous motor. A light stylus was fastened to the free end (or any position behind the tip) of a rubber sample; the other end of the sample was clamped. The elongation-time curves were recorded accurately and directly on the drum by the stylus. From these curves the velocity of the tip, as a function of elongation (or time), may be obtained by differentiation. Figure 1 shows elongation-time curves.

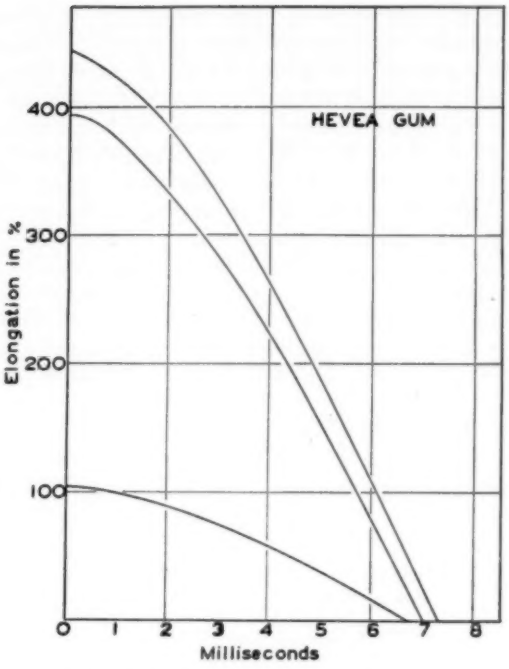

FIG. 1. Elongation-time curve for Hevea gum.

From the curve for about 450 percent initial elongation a tip velocity higher than 220 miles/hr. was derived. The velocity of the tip is sensibly constant, except for the initial acceleration at the beginning of the retraction. This initial increase is due to the weight of the stylus and of the rubber in the grip (before the sample is released).

Figure 2 gives the result of a two-stylus experiment, one at the tip, the other at the middle. It clearly shows the lag of the middle behind the tip and hence the element by element nature of the retraction. The middle moves only slightly slower than the tip.

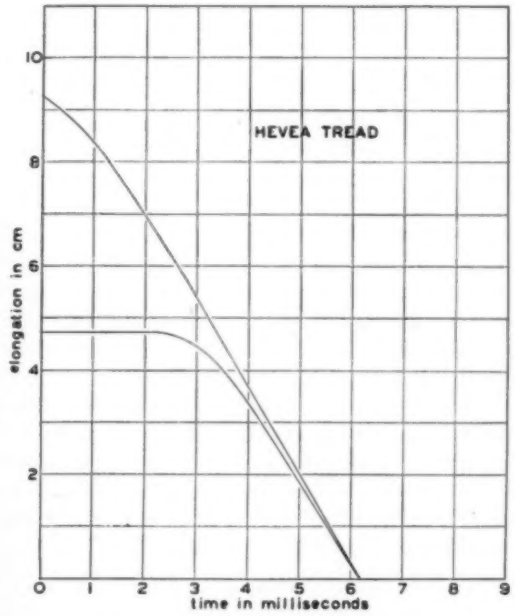

FIG. 2. Elongation vs. time with two styluses, one at the tip and one at the middle of the sample of Hevea tread.

Figure 3 shows the result of a two-stylus experiment for a Butyl tread stock. Here, the elongation-time curve for the tip has an experimental tail. The lag of the middle behind the tip is much smaller. The velocity of the middle point is also considerably smaller than that of the tip. The velocity of each section of the sample appears to be proportional to the distance from the fixed end.

Figure 4 shows the maximum tip velocities plotted against the initial elongation for all these stocks. It is noteworthy that GR-S tread, while more sluggish at small elongations, compares favorably with Hevea tread above

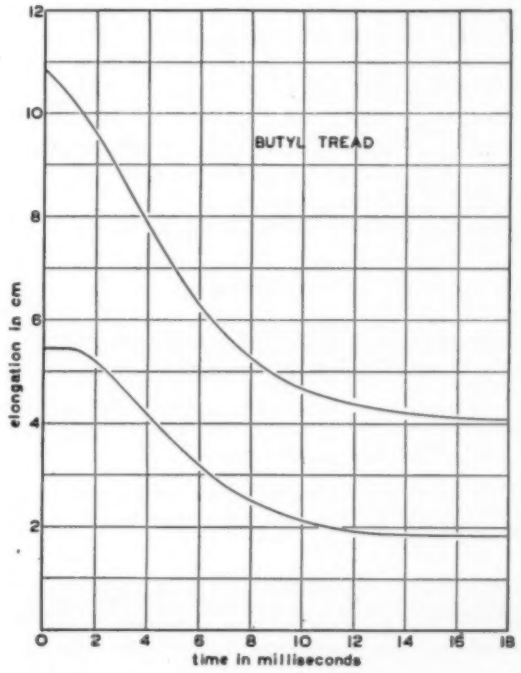

FIG. 3. Elongation vs. time with two styluses, one at the tip and one at the middle of the sample of Butyl tread.

90 percent. The great reinforcement for GR-S and the comparatively small one for Butyl are exhibited clearly in this graph.

Hevea gum and Butyl tread are the two extremes. Hevea tread, GR-S gum and tread, and Butyl gum were found to show an intermediate behavior.

The present work is the first (to the authors' knowledge) to record quantitatively the complete process of the retraction of rubber. However, early work by Exner[1] (using a method by Stefan) is equivalent to a determination of a certain average velocity of the tip of a retracting rubber sample. Exner measured the velocity of the wave pulse, which is simply connected with the velocity of the tip for materials with small internal friction like Hevea rubber.[2]

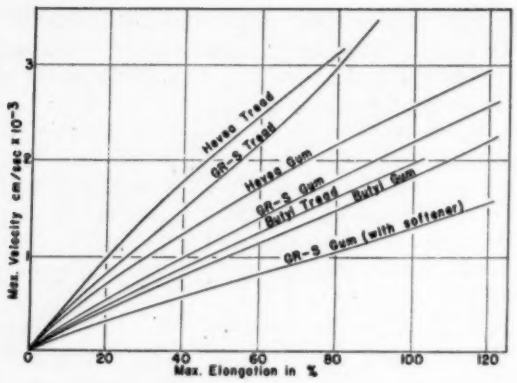

FIG. 4. Maximum velocity of tip *vs.* initial elongation for various stocks.

Simultaneously with our work, but independently, an interesting method to measure an average velocity of the tip of a sample, over a certain part of the retraction, was developed at the Goodyear Research Laboratories.[3] Whereas our method seems to be more suitable for quantitative work, the Goodyear method seems to be quite suitable for factory control purposes of stocks.

[1] F. Exner, Ann. d. Physik 153, 62 (1874).
[2] Cf. H. M. James and E. Guth, Phys. Rev. 66, 33 (1944).
[3] R. B. Stambaugh, Phys. Rev. 65, 250 (1944).

Photographic Study of the Retraction of Stressed Rubber

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July 3, 1944

THE peculiar element by element contraction of rubber, discovered by the two-stylus experiments, has been described.[1] This behavior was checked by high speed photography. Instantaneous photographs of the progress of the retraction were taken by a camera in stroboscopic illumination. Figure 1 shows a set of such photos for Hevea gum. The white marks are equidistant at the initial elongation (about 100 percent). The wave pulse and its progress are clearly visible. Figure 2 shows a set of photos for Butyl tread. Here the pulse is quite diffuse and is reflected at the fixed end before the sample has contracted completely, leading to an early buckling. Of course, the contraction starts with the tip. Figure 3 shows the differential extension, i.e., the extension of each element as a function of the distance from the fixed end for two consecutive times t and t_2, respectively. Figure 3 was plotted from the photos in Fig. 1 and Fig. 2. For Hevea gum, the progress of the well-defined wave pulse is revealed again. For Butyl tread the velocity of each point is approximately proportional to its distance from the fixed end.

Instantaneous photos do not give a time scale. Of course, the timing may be obtained from our elongation-time curves.[1] Moreover, the timing may be obtained by using

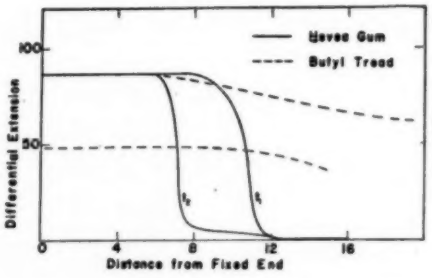

FIG. 3. Differential extension as obtained from stroboscopic photographs.

FIG. 1. Stroboscopic photographs showing retraction of Hevea gum.

FIG. 2. Stroboscopic photographs showing retraction of Butyl tread.

Edgerton's high speed camera. We have obtained high speed movies with timing; the time was recorded on the film by a spark at 1/120 second intervals. These movies check well the timing obtained by our stylus experiments, namely, that the duration of the retraction for about 100 percent initial elongation is from 5 to 20 milliseconds according to the particular sample used. After the discovery of the element by element contraction by our two-stylus experiments, instantaneous photos were taken about simultaneously at the Goodyear Research Laboratories.[2]

A complete theoretical explanation of our findings is offered in the following letter by James and Guth.[3] Hevea gum and Butyl tread represent the two limiting cases of small and strong internal friction, respectively. The expression for the velocity of the tip: $v_t = \epsilon(E/\rho)^{\frac{1}{2}}$ agrees[2] rather well with experiment.

[1] B. A. Mrowca, S. L. Dart, and E. Guth, Phys. Rev. 66, 30 (1944).
[2] R. B. Stambaugh, Phys. Rev. 65, 250 (1944).
[3] H. M. James and E. Guth, Phys. Rev. 66, 33 (1944).

Theory of the Retraction of Stressed Rubber

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AND

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THE striking behavior of stressed rubber in free retraction reported by B. Adalbert Mrowca, S. Leonard Dart, and Eugene Guth[1] can be explained by the application of the theory of stress-wave propagation in elastic solids.

We consider a uniform elastic rod of length l, fixed at one end, with linear density ρ in the unstretched state. We assume that Hooke's law holds, with Young's modulus E, and that frictional forces are proportional to the rate of deformation. Let x be a measure of distance in the material of the unstretched rod, starting at the fixed end, and let $u(x)$ be the displacement of the material corresponding to the indicated value of x. In the absence of body forces, the behavior of the rod in extension and retraction is governed by the wave equation

$$\rho(\partial^2u/\partial t^2)=f(\partial^3u/\partial x^2\partial t)+E(\partial^2u/\partial x^2). \tag{1}$$

If the rod is initially uniformly stretched with fractional extension ϵ, and is released at time $t=0$, the boundary conditions are:

$$\begin{aligned} \partial u/\partial x=\epsilon, \quad \partial u/\partial t=0 &\quad \text{for all } x \text{ at } t=0,\\ u=0 &\quad \text{for all } t \text{ at } x=0,\\ f(\partial^2u/\partial x\partial t)+E(\partial u/\partial x)=0 &\quad \text{for } x=l \text{ when } t>0. \end{aligned} \tag{2}$$

In the limiting case of small friction, $f=0$, the solution becomes, with $v=(E/\rho)^{\frac{1}{2}}$,

$$\begin{aligned} u(x,t)&=\epsilon x \qquad l-x>vt,\\ u(x,t)&=\epsilon(l-vt) \quad l-x<vt<l. \end{aligned} \tag{3}$$

According to this solution a pulse of acceleration arises at the free end at the moment it is released, and travels, with respect to the unstretched material, at a rate v; its rate of transmission in space is $v_p=(\epsilon+1)v$. The velocity of the tip of the bar is $v_t=-\partial u/\partial t=\epsilon v$. Each portion of the bar is stationary until it is reached by this pulse. As the pulse passes over it, the stress is completely relaxed, and the material reaches its final velocity ϵv. When the acceleration pulse reaches the fixed end of the material, after time $t_c=l/v$, the bar is unstressed and in uniform motion at the rate ϵv; the original potential energy is completely converted into kinetic energy. After this time a solution other than Eq. (3) begins to apply. The wave pulse is reflected at the fixed end and a pulse of deceleration begins to progress back down the bar. In the case of a rod in which transverse motion is not prevented, as in the case of a retracting rubber band, buckling begins at the moment of reflection of the pulse.

Deviations from Hooke's law cause broadening of the contraction pulse. Consideration of momentum and energy transfer makes it evident at once that an acceleration pulse of fixed form and limited extent cannot occur except for a Hooke's law bar.

In the case of high internal friction the inertia term in Eq. (1) may be neglected. The solution then becomes

$$u(x,t)\cong\epsilon x\exp\left[-(E/f)t\right]. \tag{4}$$

Equation (4) holds after the wave passed down the sample; there is a discontinuity at $t=0$.

For treatments of the intermediate case, forced vibrations, and the effect of a mass loading at the free end, reference must be made to a full treatment of the problem which is to follow.

[1] B. A. Mrowca, S. L. Dart, and E. Guth, Phys. Rev. 66, 30 (1944).

Proceedings of the American Physical Society

MEETING AT ROCHESTER, NEW YORK, JUNE 23–24, 1944

THE 261st meeting of the American Physical Society, being the 1944 Summer Meeting in the East, was held in Rochester (New York) on Friday and Saturday, June 23 and 24. Our meeting was held conjointly with one of the American Association of Physics Teachers, and comprised the inaugural meeting of our own Division of High Polymer Physics. To that Association and this Division belongs most of the credit for arranging an excellent threefold meeting, with 238 registrants and 180 in attendance at the dinner. The Association had a symposium, "Essential physics for a naval career," and another, "The training of physicists for industry," in which we were graciously allowed to join as co-sponsors; it had also two programmes of contributed papers. We had two programmes of contributed papers (the abstracts follow) and an invited paper, "Ultraviolet spectra of biologically important molecules," by G. A. Anslow. The host institution was the University of Rochester, and to Professor F. C. Fairbanks and his department and secretarial staff warm thanks are expressed. The weather was merciful for the season. The dinner of Friday evening at the Hotel Seneca was followed by brief speeches by A. J. Dempster, L. McDowell, J. H. Dillon, and L. A. DuBridge, and an address, "The new world and the scientist," by C. E. K. Mees.

The Division of High Polymer Physics of our Society made its debut with an inaugural programme of four scientific sessions comprising twenty-one invited papers. The titles of these papers are printed below; abstracts and in some cases complete texts are to be published elsewhere. This programme was arranged by J. H. Dillon, M. L. Huggins, W. J. Lyons, and H. F. Mark, who with W. F. Busse served as chairmen. A draft of by-laws was evolved and adopted by the Division in business session and by the Council of the Society, and in accordance with its stipulations the Council appointed an initial executive committee of seven members to serve until the 1944 Annual Meeting of the Society. These seven are: R. B. Barnes, W. F. Busse, P. Debye, J. H. Dillon, J. P. Elting, W. J. Lyons, and L. A. Wood. The Division of High Polymer Physics is now launched and fully equipped and ready to proceed under its own steam. Communications should be addressed to its Secretary Dr. W. J. Lyons, 2100 Robert E. Lee Boulevard, New Orleans 19, Louisiana.

Other actions by the Council at its meetings of Friday and Saturday included the appointment of a "steering committee" of three members (K. K. Darrow, A. J. Dempster, R. S. Mulliken) to guide the formation of the Division of Spectroscopy authorized at the April meeting, with power to add other members to the Committee and to arrange symposia; and the appointment of a committee of six (S. Dushman, T. A. Read, F. Seitz, W. Shockley, S. Siegel, R. Smoluchowski) to arrange a symposium and take other steps to promote a Division of Physics of the Solid State.

The Council elected four candidates to Fellowship (one had previously been a Member) and fifty-five candidates to Membership; their names are subjoined.

The Society has lost through death four of its Fellows: S. Ballantine (Ballantine Laboratories), N. A. Kent (Boston University), Margaret E. Maltby (Barnard College), and O. M. Stewart (University of Missouri).

Elected to Fellowship: J. Hillier, H. F. Mark, E. Ott, E. M. Purcell.

Elected to Membership: Bartnoff, Shepard; Barton, David Maxwell; Brody, Selma Blazer; Brown, Barremore Beverly; Budd, Chester B.; Burns, Preston; Claassen, Howard H.; Clark, Orrin H.; Ellis, James; Ferguson, Jasper N.; Gibson, Kenneth Alvin; Gilbert, Edgar N.; Graves, J. Dentler; Halteman, Eber K.; Harth, Erich M.; Hook, Alonzo Lohr; Jastrow, Robert; Kaufmann, Stefan G.; Kessler, Myer; Kinnaird, Richard F.; Labin, Edouard; Lanzl, Lawrence H.; Levine, Samuel; Lewin, Gerhard; Lopes, Jose Leite; Madansky, Leon; Maiden, Robert M.; Mathieson, Raymond L.; McCarthy, Edward L.; Munier, John Hammond; Pardee, Otway O'Meara; Perkin, Richard S.; Pleasonton, Frances; Plew, Herman E., Jr.; Prather, Robert W.; Proskauer, Eric S.; Rall, Waldo; Rich, Joseph Anthony; Schwartz, Don L.; Schwarz, Edward Robinson; Sharer, William W.; Shaw, Morton R., Jr.; Sreb, Jules H.; Stephenson, John; Strong, Guy E.; Tallmadge, Francis K.; Thomas, Benjamin W.; Vogel, Charles P.; Weltmann, Ruth N.; Wetmore, Warren L.; White, Donald W., Jr.; Woods, Frank R., Jr.; Wundheiler, Alexander; Urquhart, Robert Cameron; Ziomek, Joseph S.

KARL K. DARROW
Secretary

Symposia of the American Association of Physics Teachers and the American Physical Society

Essential Physics for a Naval Career

Introduction. COMMANDER F. K. ELDER, USN (Ret.), *Head of Physics Committee, U.S.N.A.*
Vectors. LIEUTENANT COMMANDER J. A. TIEDEMAN, USNR, *Instructor in Physics, U.S.N.A.*
Mechanics. LIEUTENANT COMMANDER L. E. KINSLER, USNR, *Instructor in Physics, U.S.N.A.*
Gyroscopes. LIEUTENANT COMMANDER J. D. RIGGIN, USNR, *Instructor in Physics, U.S.N.A.*
Sound. LIEUTENANT E. R. PINKSTON, USNR, *Instructor in Physics, U.S.N.A.*
Optics. LIEUTENANT (J.G.) R. A. GOODWIN, USNR, *Instructor in Physics, U.S.N.A.*

The Training of Physicists for Industry

Postwar Training of Physicists for Industry. SAUL DUSHMAN, *General Electric Company.*
Training for the Small Industrial Research Laboratory. H. L. MASON, *Taylor Instrument Companies.*
Training of Physicists for the Optical Industry. BRIAN O'BRIEN, *University of Rochester.*
Observations on the Training of Physicists. M. N. STATES, *Central Scientific Company.*
Discussion, opened by J. T. LITTLETON, *Corning Glass Company.*

Invited Papers of the Inaugural Programme of the Division of High Polymer Physics

Systems with Superimposed Viscous and Elastic Behavior. A. TOBOLSKY, *Princeton University.*
Transition Phenomena in High Polymers. W. O. BAKER, *Bell Telephone Laboratories.*
A Contribution to the Kinetic Theory of Rubber Elasticity. T. ALFREY, *Monsanto Chemical Company.*
A Recent Development in the Theory of Rubber Elasticity. HUBERT M. JAMES AND E. GUTH, *University of Notre Dame.*
Electrostatic Properties of Rubber and GR-S. R. S. HAVENHILL, H. C. O'BRIEN, AND J. J. RANKIN, *St. Joseph Lead Company.*
Speed of Retraction of Rubber. R. B. STAMBAUGH, MARGARET ROHNER, AND S. D. GEHMAN, *Goodyear Tire & Rubber Company.*
Rise of Temperature on Fast Stretching of Butyl Rubber. S. L. DART AND E. GUTH, *University of Notre Dame.*
Limiting Law of the Reinforcement of Rubber. HUGH M. SMALLWOOD, *U. S. Rubber Company.*
Relations Between Stress, Strain, and Temperature in a Pure-Gum Vulcanizate of GR-S. FRANK L. ROTH AND LAWRENCE A. WOOD, *National Bureau of Standards.*
Some Low Temperature Properties of Elastomers. F. S. CONANT AND J. W. LISKA, *Firestone Tire & Rubber Company.*
Physical Properties of Elastomers at Low Temperatures. T. L. LOUGHBOROUGH, H. E. GREENE, AND C. W. HARRIS, *B. F. Goodrich Company.*
Molecular Weights and Weight Distribution of High Polymers Determined from Light Scattering. P. M. DOTY, B. H. ZIMM, AND H. MARK, *Polytechnic Institute of Brooklyn.*
Theory of Filler Reinforcement. E. GUTH AND HUBERT M. JAMES, *University of Notre Dame.*
Some Factors Influencing the Brittle Point of High Polymers. R. S. SPENCER AND R. F. BOYER, *The Dow Chemical Company.*
Kinematic Errors in Bobbin Drives on Roving Frames. S. L. GERHARD, *United States Rubber Company.*
Control of Elongation in Highly Stretched Tire Cord. H. J. PHILIPP AND C. M. CONRAD, *Southern Regional Research Laboratory, U. S. Department of Agriculture.*
The Influence of Velocity-Gradient on the Relation Between Viscosity and Concentration in Cuprammonium Solutions of Cellulose. W. J. LYONS, *Southern Regional Research Laboratory, U. S. Department of Agriculture.*
Orientation of Hydrophile Xerogels in an Electric Field, Part II. S. E. SHEPPARD AND P. T. NEWSOME, *Eastman Kodak Company.*
Determination of Molecular Weights of High Polymers by Solubility Methods. D. R. MOREY AND J. W. TAMBLYN, *Eastman Kodak Company.*
Comparison of the Structures of Stretched Linear Polymers. M. L. HUGGINS, *Eastman Kodak Company.*
Fractionation, Viscosity, and Osmotic Pressure Studies on Cellulose Esters. J. W. TAMBLYN, R. L. TICHENER, D. R. MOREY, AND R. H. WAGNER, *Eastman Kodak Company.*

Abstracts of Contributed Papers

A1. X-Ray Coincidences From Te^{121} (125 Days).* ROSALYN S. YALOW AND M. GOLDHABER, *University of Illinois.*—Te^{121} (125d) which decays by K-electron capture into Sb^{121} can be produced by deuteron bombardment of antimony-The radiations emitted by Te^{121} are very complex, consist. ing of x-rays, internal conversion electrons, and γ-rays. By critical absorption and fluorescence studies, O'Neal and Scharff-Goldhaber found not only the $K\alpha$ lines of Sb but also x-rays of somewhat shorter wave-length ("tellurium like"). They interpreted these x-rays as due to a transition in the Sb atom after both K electrons had been removed, one by K-electron capture and the other by a succeeding internal conversion of a γ-ray. We have obtained further evidence strengthening this interpretation by observing Geiger counter coincidences between the x-rays emitted during the decay of Te^{121}. The X-X coincidence rate observed was about 10^{-3} per x-ray counted in a single counter. When the x-ray rate in a single counter is reduced to about half by interposing different absorbers of suitable thickness between the Te sample and the counters, it is found that Cd foils, which absorb preferentially the shorter wave-length member of the x-ray pair, reduce the coincidence rate more strongly than either Ag or Al foils. From the relative intensity of the "double hole" x-ray transition to that of the "single hole" x-ray transition, one may obtain an estimate of the absolute mean lifetime of internally converted nuclear excited states by using for comparison the mean lifetime for which a hole in the K-electron shell exists. This time can be calculated from the known widths of x-ray lines. One may conclude that the excited state of the Sb^{121} nucleus which gives rise to strong internal conversion lines has a mean lifetime $\lesssim 2\times10^{-17}$ sec.

* Read by title.

A2. Slowing Down of R Neutrons into C Neutrons in Lead. C. O. MUEHLHAUSE AND M. GOLDHABER, *University of Illinois.*—When fast neutrons are slowed down inside paraffin they pass through an energy region above the Cd absorption limit where they may be referred to as R neutrons. After a few further collisions with protons they become C neutrons. It is of interest to study with what probability R neutrons can be slowed down into C neutrons by interaction with heavy elements. A previous attempt to detect this transformation was unsuccessful. With a more sensitive arrangement we have now been able to detect such an effect in lead and other elements. Its magnitude depends to some extent on geometrical and other factors. With one arrangement the slowing down effect from R into C neutrons per lead atom was found to be 5 percent of that observed per hydrogen atom. It is unlikely that more than a small part of the effect is due to hydrogen contamination in the lead. The effect appears to be too large, however, to be accounted for by excitation of lattice vibrations alone. No effect was observed with our arrangement with either Ra-α-Be or Ra-γ-Be sources when no paraffin was present to slow down the primary neutrons into the R region.

A3. Nuclear Excitation of Cadmium. MARCELLUS L. WIEDENBECK, *University of Notre Dame.*—By x-ray irradiation of self-quenching argon-ether counters having cadmium cathodes, the excitation curve for this element was obtained up to 2.9 mev. The half-life[1,2] was found to be 48.7 ± 0.3 minutes. The threshold for the excitation is 1.25 ±0.03 Mev and activation levels were found at 1.68 Mev, 2.08 Mev, and 2.56 Mev. The electron excitation curve shows the same threshold and activation levels which were found by x-ray excitation. Absorption of the conversion electrons by aluminum foils indicates that the energy of the metastable state is 195 kv.

[1] Dode and Pontecorvo, Comptes rendus **207**, 287 (1938).
[2] J. R. Feldmeier and G. B. Collins, Phys. Rev. **59**, 937 (1941).

A4. The Electron Affinity of Bromine and a Study of Its Decomposition on Hot Tungsten. P. M. DOTY AND J. E. MAYER, *Columbia University.*—The electron affinity of bromine has been measured by determining the ratio of ions to electrons leaving a hot tungsten surface in contact with bromine gas at low pressure. The value obtained is 80.5 ± 0.4 kcal./mole. The measurements, which extend over a range of 700° in temperature and 80-fold in pressure, show no trend in the electron affinity and demonstrate that the decomposition of bromine under these conditions is a first-order reaction. The assumption of unit accommodation coefficient is shown to be correct. The agreement between the value here obtained and the value calculated from energy cycles substantiates the improved method of calculating lattice energies.

A5. Analysis of the Raman Spectrum of a New C_5H_8 Compound. FORREST F. CLEVELAND AND M. J. MURRAY, *Illinois Institute of Technology.*—Murray and Stevenson[1] recently reported Raman frequencies and relative intensities for a new C_5H_8 compound and presented five possible structures for the molecule. The method of preparation as well as the values of the C−H stretching frequencies indicated that the compound was spiropentane. In the present investigation, the depolarization factors of the Raman lines have been obtained and the expected number of polarized and depolarized frequencies, both for the entire molecule and for the carbon skeleton ($\Delta\nu$ less than 1050 cm^{-1}), have been worked out for each of the five structures by use of the group theory formulas. If it is assumed that all lines involved in accidental degeneracies have been resolved, the structure giving the best correlation with the experimental results is one having the symmetry D_{2h}. However, if unresolved accidental degeneracies do exist, as seems quite probable for the spiropentane structure since there are a number of similar groups in the molecule, the spiropentane structure of symmetry C_{2v} would give the best correlation. A calculation of the frequencies would aid in deciding between the two structures. The large number of approximate coincidences between the Raman and infra-red frequencies is additional evidence in favor of the spiropentane structure.

[1] Murray and Stevenson, J. Am. Chem. Soc. **66**, 314 (1944).

A6. The Formation of a Zone Plate by Newton's Rings. ROBERT AYRES WOODSON, *Eastman Kodak Company.*—The approximate equation for the radius y' of the Nth ring of a zone plate is: $y'^2 = Nf\lambda = Ny_1'^2$, where f = focal length, λ = wave-length, y_1' = radius of first ring. The approximate equation for the radius y'' of the Nth Newton ring is: $y''^2 = N\lambda/n(C_1 - C_2) = Ny_1''^2$, where λ = wave-length, n = index of refraction of thin film, C_1 = curvature of one surface of thin film, C_2 = curvature of other surface of thin film, y_1'' = radius of first ring. Since the radius of the Nth ring is proportional to $\sqrt{N}$ for both the zone plate and Newton's rings, it occurred to the author to use Newton's rings for the formation of a zone plate. On combining the above equations, we obtain for the power P of such a zone plate, $P = n(C_1 - C_2)$, or for an air film between the lens surfaces, $P = C_1 - C_2$. For the combination of a plano-parallel plate and the convex surface of a zero-power meniscus lens, the focal length is equal to the radius of the convex surface.

A7. Some Basic Thermodynamic Considerations. J. L. FINCK, *College of the City of New York.*—In previous papers the writer has endeavored to show that from a phenomenological point of view there is justification for including metastable states within classic thermodynamics, particularly if catalytic phenomena are considered. In doing so, a much broader view may be taken of the thermodynamic behavior of systems. However, criteria based exclusively on the first two laws of thermodynamics define equilibrium in such manner that metastable states are excluded. In the present paper a critical examination is made of development of the second law, and a fallacy is found wherein the metastable states are excluded from the very start—in the treatment of Carnot's cycle. Logical approach requires that we establish the independent variables in advance, and it is found that in general an equation of state should include more than two independent variables. Such an equation will include metastable, as well as ordinary equilibrium states, and will apply to systems consisting of one or several phases. In enlarged systems it is shown that entropy is no longer an integrable function, and therefore cannot play the role it does in classic thermodynamics. It is suggested how an enlarged equation of state may explain thermodynamically the catalytic behavior of systems where intermediate products are formed.

B1. Knudsen Type Vacuum Gauge in Technical High Vacuum Work. A. F. TURNER AND O. A. ULLRICH, JR., *Bausch & Lomb Optical Company.*—The Knudsen vacuum gauge has not found widespread application. Apparently the principal objection hitherto has been the need for a vibration-free support. This restriction can be materially relaxed in the design of DuMond and Pickels[1] by the addition of a small auxiliary internal damping vane moving in a horizontal plane between permanent magnets. External permanent magnets damp the impact vane. Gauges so constructed and mounted rigidly on the base plates of large bell jar systems have given satisfactory performance in evaporation work under adverse floor vibration conditions. The gauge is practically linear through the 10^{-6} mm Hg range using a sensitivity of 1×10^{-6} mm Hg per cm deflection at 23-cm scale distance. Operation in higher and lower ranges is obtained by changing the heater temperature. The gauge is not harmed if accidentally turned on at atmospheric pressure—a definite advantage in a technical instrument. The performance of the gauge during the evaporation of some metals and salts will be described.

[1] Jesse W. M. DuMond and W. M. Pickels, Jr., Rev. Sci. Inst. 6, 362 (1935).

B2. Solution of the Diffusion Equation Applicable to the Edgewise Growth of Pearlite. W. H. BRANDT, *Westinghouse Electric & Manufacturing Company.*—The diffusion equation is transformed to a set of coordinates moving with the pearlite interface and a solution applicable to the problem obtained in the form of an infinite series of terms. Using the first three terms, the edgewise velocity of pearlite growth is calculated for a plain carbon eutectoid steel using data most of which are obtained by extrapolation. The values obtained show reasonable agreement with values for the rate of pearlite nodule growth determined by Hull, Colton, and Mehl.* The velocity increases with decreasing temperature, as expected, and it is shown that this is due to the change in the solubility of ferrite and cementite in austenite with temperature. The theory predicts curved ferrite and cementite interfaces and the carbon concentration in austenite is shown to vary across each of these interfaces.

* F. C. Hull, R. A. Colton, and R. F. Mehl, Trans. A.I.M.E. 150, 185 (1942).

B3. The Mechanics of the Metal Cutting Process. II. Oblique Straight-Edged Tool and Continuous Chip. M. EUGENE MERCHANT, *Cincinnati Milling Machine Company.*—The principles employed in the derivation of the mechanics of the simple case previously discussed[1] have been extended to an analysis of the geometry of chip formation and of the force system occurring in the more general case of a straight-edged tool with cutting edge oblique to the direction of relative motion of tool and workpiece. It is found that the direction of chip flow and the direction of shear are no longer perpendicular to the cutting edge as they are in the simple case. As a result the force system is three-dimensional. However, it is again possible to obtain simple relationships between the readily measurable force components and geometrical variables, and such basic quantities as strain in the chip, friction between chip and tool, shear strength of the metal, etc., thus making possible the analysis of actual machining operations in terms of these basic quantities.

[1] M. E. Merchant, Bull. Am. Phys. Soc. 19, No. 2, 7 (1944).

B4. Experimental Investigations of the Mechanism of Friction. B. W. SAKMANN, *Massachusetts Institute of Technology.*—In all friction experiments material is exchanged between the sliding surfaces even under small loads of a few grams. This fact seems to indicate that such an exchange of matter might be of fundamental importance in all frictional phenomena. The development of a radioactive tracer method made it possible to investigate this transfer of matter. Spherical riders were slid over an activated base surface and were tested for the presence of radioactive material. The amount of material transferred depends upon various parameters. The amount of base metal adhering to

the rider was found to be proportional to the normal load and the distance traveled. The transferred material increased with the surface roughness if the rider was harder than the base. If, on the other hand, the rider was softer than the base, surface finish was of minor importance. For steel samples the deposited material was inversely proportional to the Brinell hardness. For riders of different metals of the same hardness and surface finish the amount of transferred material increased with the solid solubility of the base metal in that of the rider. The exchange of metal was reduced by lubrication. The reduction depended on the normal load and, for the same load, on the material of the rider.

B5. Tensile Strength of Liquids. ALLEN KING, *Dartmouth College.*—When a tension is applied *slowly* to a liquid column, rupture occurs with the formation of a cavity. The critical tension for rupture probably fluctuates about a mean. The smallest cavity in which vapor can exist in equilibrium within a liquid at absolute temperature T may be assumed to have a diameter equal to the average distance between molecules of vapor at the vapor pressure for this temperature. This cavity is in unstable equilibrium. If q represents the critical tension (tensile strength) in atmospheres, then $q=14.6\gamma(p/T)^{\frac{1}{3}}$, where γ is the free surface energy in ergs per cm^2 and p is the vapor pressure in cm of Hg. At 20°C q has a value of 194 atmos. for water, 80 atmos. for ethyl alcohol, and 132 atmos. for diethyl ether. According to this relation q increases with temperature to a maximum value.

B6. Ponderomotive Forces on Matter in Electric and Magnetic Fields. FELIX EHRENHAFT, *New York City.*—In a recent Letter to the Editor of *The Physical Review* it was stated that beside the force of gravity and the magnetic action of electric currents (Oersted, Biot-Savart, H. A. Lorentz), $f_e=D+1/c[v_e, H]$, there is a third general force, the electric action of magnetic currents, $f_m=B+1/c[v_m, E]$. However, if one adds to the electrodynamic equations the term of the magnetic current, following the conception of Stefan and Heaviside, it seems that the forces f_e and f_m cannot explain all the observed movements. If there is no angle between the direction of the electric current and the magnetic field, and no angle between the direction of the magnetic current and the electric field, the second terms of the above equations become nil. But it has been observed that, when the homogeneous electric and magnetic fields are parallel and in action simultaneously, there can be a circular movement of matter. One suspects that some of the phenomena reported by S. P. Thompson and others are of the same type, although the homogeneity of the fields they employed was not considered by them. Experiments in gases as well as in liquids will be described where these movements are to be seen in homogeneous fields. Conclusions can only be drawn under the preliminary assumption that there is no interdependency between H and E other than that expressed in the equations mentioned above.

Read by Title

T1. The Electrical Resistivity of Alpha-Phase Copper-Zinc Alloys at Low Temperatures. H. A. FAIRBANK, *Yale University.*—A systematic investigation of the resistivity (ρ) of the copper-zinc alpha-phase solid solution system has been made in the temperature range 14.3°K to room temperature. Measurements were taken on two sets of one millimeter diameter wires. The first was annealed at 700°C and quenched; the second, cold-drawn through a 75 percent reduction and left in the hard-drawn state. For the annealed set, the resistivity varied linearly with the temperature above 70°K and with a higher power of T at low temperatures, approaching a constant value below 20°K. Above 70°K, $d\rho/dT$ was found to increase linearly with atomic percent solute in variance with Matthiessen's rule. The residual resistivity increased with percent added zinc, but not in agreement with Nordheim's theory,[1] unless it be assumed that the effective number of free electrons per atom increases with added solute atoms or that the specimens were not true random solid solutions. The hard-drawn specimens showed properties similar to the annealed group except for an increase in the resistivity due to residual strains.

[1] L. Nordheim, Ann. d. Physik 9, 664 (1931).

T2. The Electrical Resistivity of Alpha-Phase Copper-Tin Alloys at Low Temperatures. H. A. FAIRBANK, *Yale University.*—An investigation similar to that described in the previous abstract for the Cu-Zn alpha-phase alloys has been carried out for the Cu-Sn alpha-phase alloy system in the temperature range 14.3°K to room temperature. The dependence of the electrical resistivity on temperature and on percent added tin was similar to that found for the Cu-Zn system. However, the cold work produced some unexpected effects. Although the expected increase of resistivity with residual strain occurred in the higher percentage tin specimens, the wires of zero to three atomic percent tin, which were hard-drawn through a 75 percent reduction and left in the strained state, showed a lower resistivity at low temperatures than did the corresponding annealed, quenched specimens. This anomaly increased as the temperature was lowered. This unusual behavior may be due to order in the Cu-Sn specimens of about two atomic percent tin, induced either during the cold working process or during the air cooling which followed the anneal at 750°C prior to the 75 percent reduction. It is also possible that a phase separation may have occurred as a result of the extensive cold work. Further investigation appears highly desirable.